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Review Concept of Application of Differential Equations

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Abstract: This article attempts the basic concepts of Differential equations and studies one of the application of Differential equations which is Newton's law of cooling. It gives the derivation of this and explains its role in Engineering connection. It solves some problems related to this.
Keywords: Differential equation, heat flow, Newton's law of cooling

1.INTRODUCTION:

One physical system in which many important phenomena occur is that where an initial uneven temperature distribution causes heat to flow.As heat flows the temperature distribution changes ,which modifies the heat flow. That is heat flows from hot places to cold ones,and as this happens the temperature of the cold places rises and the temperature of hot places decreases.

There are two rough mathematical rules governing the relation between heat flow and temperature change.Heat flow is proportional to rates of change in temperature distribution.The time of temperature change at any point is proportional to the rate of heat flow into that point

2. PRELIMINARIES AND BASICS:

2.1 Definition: A differential equation is an equation that relates an unknown function and one or more of its derivatives of with respect to one or more independent variables. For instance, the equation $\frac{dy}{dx} = -5x$ relates the first derivative of y with respect to x, with x. Here x is the independent variable and y is the unknown function (dependent variable).

If $\frac{d^n y}{dx^n}$ is the n-th derivative of y with respect to x. One can also use the notation y (n) to denote $\frac{d^n y}{dx^n}$. It is further convenient to write y ′ = y (1) and y ″ = y (2). In physics, the notation involving dots is also common, such that $y^{\cdot}$ denotes the first-order derivative.

Different classifications of differential equations are possible and such classifications make the analysis of the equations more systematic.

2.2 Classification of Differential Equations

There are various classifications for differential equations. Such classifications provide remarkably simple ways of finding the solutions (if they exist) for a differential equation. Differential equations can be of the following classes.

2.2.1 Ordinary Differential Equations

If the unknown function depends only on a single independent variable, such a differential equation is ordinary.

The following is an ordinary differential equation: $L\frac{d^2}{dt^2}Q(t) + R\frac{d}{dt^2}Q(t) + \frac{1}{C}Q(t) = E(t),$, which is an equation which arises in electrical circuits. Here, the independent variable is t.

2.2.2 Partial Differential Equations

If the unknown function depends on more than one independent variables, such a differential equation is said to be partial. Heat equation is an example for partial differential equations: $\alpha^2 \frac{\partial^2}{\partial x^2} f(x,t) = \frac{\partial}{\partial t} f(x,t)$ Here x, t are independent variables.

2.2.3 Homogeneous Differential Equations

If a differential equation involves terms all of which contain the unknown function itself, or the derivatives of the unknown function, such an equation is homogeneous. Otherwise, it is non-homogeneous.

2.2.4 N-th order Differential Equations

The order of an ordinary differential equation is the order of the highest derivative that appears in the equation. For example 2y (2) + 3y (1) = 5, is a second-order equation.

2.2.5 Linear Differential Equations

A very important class of differential equations are linear differential equations. A differential equation F(y, y(1), y(2), . . . , y(n))(x) = g(x) is said to be linear if F is a linear function of the variables y, y(1), y(2), . . . , y(n)

2.3 Note: If a differential equation is not linear, it is said to be non-linear.

2.4 Solutions of Differential equations

2.4.1 Definition

To say that y = g(x) is an explicit solution of a differential equation F(x, y, dy/ dx, d ^{n}y/ dx^n) = 0 on an interval I ⊂ R, means that F(x, g(x), dg(x) dx , . . . , d ng(x) dxn) = 0, for every choice of x in the interval I.

2.4.2Definition

We say that the relation G(x, y) = 0 is an implicit solution of a differential equation F(x, y, dy/ dx, d ^{n}y/ dx^n) = 0 if for all y = g(x) such that G(x, g(x)) = 0, g(x) is an explicit solution to the differential equation on I.

2.4.3 Definition

An n-th parameter family of functions defined on some interval I by the relation h(x, y, c_1, . . . , c_n) = 0, is called a general solution of the differential equation if any explicit solution is a member of the family. Each element of the general solution is a particular solution.

2.4.4 Definition

A particular solution is imposed by supplementary conditions that accompany the differential equations. If all supplementary conditions relate to a single point, then the condition is called an initial condition. If the conditions are to be satisfied by two or more points, they are called boundary conditions. Recall that in class we used the falling object example to see that without a characterization of the initial condition (initial velocity of the falling object), there exist infinitely many solutions. Hence, the initial condition leads to a particular solution, whereas the absence of an initial condition leads to a general solution.

2.4.5Definition

A differential equation together with an initial condition (boundary conditions) is called an initial value problem (boundary value problem).

2.5 Basic definitions

- Order: Highest Derivative present in the differential equation
- Degree: Power of the higher derivative after free from radicals and fractions
- Formation of Differential Equation: 1. Consider the given equation f(x,y,a,b,c…d)=0
  2. Observe the no. of arbitrary constants if it is 'n'
  3. Based on Arbitrary constants differentiate the given Equation n times
  4. Using the n+1 equations including the givenequation We get a differential equation of nth order.
- The standard form of First order linear differential equation is of the form $\frac{dy}{dx} = f(x,y)$

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Homogeneous Function: A function f(x,y) which satisfies the condition that

$$f(kx, ky) = k^n f(x, y)$$

A differential equation is said to be Homogeneous if the f(x, y) is homogeneous of degree zero

For Homogeneous use $y = vx \; or \; x = vy$ substitution based on the equation

For Non-Homogeneous in the case of $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ use ax+by=t as substitution and in the case of $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ take $x = X + h \; and \; y = Y + k$ as substitution

- The necessary and sufficient condition for the differential equation M(x,y)dx+N(x,y)dy=o to be exact is $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$
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The solution of the exact differential equation is $\int Mdx + \int Ndy = c$,under the first integral y constant and under the second integral terms do not contain x

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The equation is not an exact differential equation if $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ and it can be solved by applying various conditions like homogeneous, inspection, functions in xy, function of x alone, function of y alone and powers of x,y

- The standard form of First order linear differential equation is of the form $\frac{dy}{dx} = f(x, y)$
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The linear differential equation of first order and first degree of Leibnitz is of the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

- The solution of the Leibnitz linear differential equation is $y(I.F) = \int Q(x)(I.F)dx + c$
- The Bernoulli's differential equation is of the form $\frac{dy}{dx} + P(x)y = Q(x)y^n$

  To find the solution of Bernoulli's use the substitution as $y^{1-n} = t$

2.Newton's Law of Cooling:

It states that the rate of change of temperature of a body is directly

proportional to the difference between temperature of a body and its surrounding

medium. i.e dT/dt □ (T₮Ts)

the Newton's Law of Cooling is given by

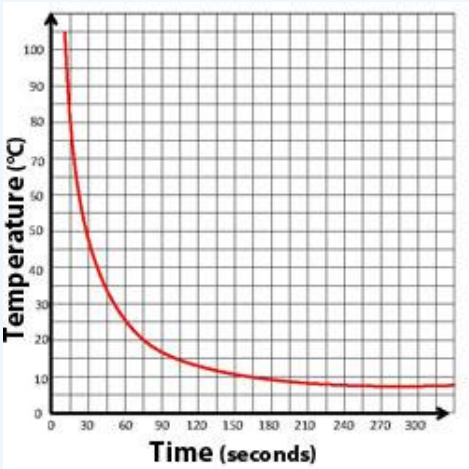
$$dT/dt = k (T_t - T_s)$$

Where T_t is the temperature at time t and
T_s is the temperature of the surrounding,
k is a constant.
The Newton s Law of Cooling Formula is given by
$T(t) = T_s + (T_0 - T_s) e^{-kt}$
Where t is the time taken,
T(t) is the temperature of the given body at time t,
T_s is the surrounding temperature,
T_o is the initial temperature of the body,
k is the constant.
NOTE:The greater the difference in temperature between the system and surrounding, more quickly the body temperature changes.

3 .Engineering Connection:

Heat transfer is a broad topic used in many branches of engineering. For example, mechanical engineers who design engines from steam locomotives to modern internal combustion engines on a detailed understanding of how heat moves through all types of matter. Industrial engineers use heat transfer concepts to design climate control systems for manufacturing facilities, such as foundries or refrigerated food production facilities, which integrate temperature-sensitive human workers with extreme temperature processes. Moreover, heat transfer is so critical to biological engineering that it has spawned the specialty of "bioheat" transfer, which is the study of normal functioning of the cardiovascular system as well as inherently heated treatments such as cryo-surgery and laser-based therapies.

For example, If you placed a room-temperature can of soda in the refrigerator and waited for it to cool, its temperature change with respect to time is given by



4. PROBLEMS

Problem 1

A thermometer which has been at 70 F inside a house is placed outside where the air is at 10 F.3 min later it is found that thermometer is at 25 F .Find the thermometer reading after 6 minutes?

Problem 2

A cup with water at 45 ^{o}C is placed in the cooler held at 5°C .If after 2 min the water temperature is 25 ^{o}C,when will the water temperature be 15° C?

Problem 3

A hard boiled egg at temperature 100° C is placed in 15° C water to cool.5 Minutes later the temp of egg is 55° C.When will the egg be 25 ^{o}C?

5.CONCLUSION:

This article explains one of the application of Differential equations which is “Newton’s law of cooling”.It gives the derivation of this and explains its role in the engineering connection.It solves some problems by using this.

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