

## Effect of Initial Stress and Magnetic Field on Shear Wave Propagation under Gravity Field

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**Abstract:** The purpose of this paper we have studied shear waves propagation in a non-homogeneous, Anisotropic incompressible initially stressed Medium with magnetic field and gravity field. The velocity of shear wave propagation depends upon the direction of propagation, the anisotropy, Gravity field, Magnetic field, non homogeneity of the medium and the initial stress have been analyzed. The frequency equation that determines the velocity of the shear wave, has been obtained. The dispersion equations have been obtained and discussed for different cases. The results have been presented graphically.

**Keywords:** Incompressible, initial-stress, anisotropic, magnetic field, Gravity field, shear-wave

### 1. Introduction

Most materials behave as incompressible media and the velocities of longitudinal waves in them are very high. The Varieties of hard rocks present in the earth are also almost incompressible. Due to the factors like external pressure, slow process of creep, difference in temperature, manufacturing processes, nitriding, pointing etc. The medium stay under high stresses. These stresses are regarded as initial stresses. Owing to the variation of elastic properties and the presence on these initial stresses, the medium becomes isotropic as well. Problem of shear waves in an orthotropic elastic medium is been very important for the possibility of its extensive application in various branches of Science and Technology, particularly in Optics, Geophysics and plasma physics.

Shear waves propagating over the surface of homogeneous and inhomogeneous elastic half-spaces are a well-known and prominent feature of waves theory. Earlier many researchers have worked on the shear-wave propagation in anisotropic media. **Acharya and Sengupta[1979]** discussed the influence of gravity field on the propagation of waves in a thermoelastic layer. **De and Sengupta[1978]** investigated many problems of elastic waves and vibration under the influence of gravity field. **Abd-Alla[1999]** studied the effect of initial stress and orthotropy on the propagation waves in a hollow cylinder. **Singh[2007]** studied wave propagation in a generalized thermoelastic material with voids. **Pal[1989]** has studied generation of SH-type waves in layered orthotropy on the propagation waves in a hollow cylinder. **Zhang and Batra[2007]** wave propagation in functionally graded materials by modified smoothed particle hydrodynamic method. **Garg[2007]** considered effect of initial stress on harmonic plane homogeneous waves in viscoelastic anisotropic media, **Vecsey, et al.[2007]** Investigated shear wave splitting as a diagnostic of variable anisotropic structure of the upper mantle beneath central Fennoscandia. **Willis and Movchan[2007]** discussed propagation of elastic energy in a general anisotropic medium. **Zhu and Shi[2008]** discussed wave propagation in non-homogeneous magneto-electro-elastic hollow cylinders. **Stuart and Sheila[2008]** investigated shear horizontal wave in transversely in homogeneous plates. **Jiangong et. al[2008]** Studied wave propagation in non-homogeneous magneto-electro-elastic plates. Rayleigh wave in a magnetoelastic initially stressed conducting medium with the gravity field are investigated by **EI-Naggare et. al [1994]**.

In this paper the study of shear waves propagation in effect of initial stress, magnetic field, non-homogeneity of the medium and gravity field in an orthotropic elastic solid medium has been discussed using the wave equations. Which satisfied by the displacement potentials. The frequency equation that determines the velocity of the shear wave has been obtained. The dispersion equations have been obtained and investigated for different cases.

### 2. Formulation of the Problem

We consider an unbounded incompressible anisotropic medium under initial stresses  $s_{11}$  and  $s_{12}$  along the x, y direction, respectively. When the medium is slightly disturbed, the incremental stresses  $s_{11}$ ,  $s_{12}$  and  $s_{22}$  are developed and the equation of motion in the incremental state given by **EI-Naggare et al.**

$$\frac{\partial s_{11}}{\partial x} + \frac{\partial s_{12}}{\partial x} - p \frac{\partial w}{\partial y} - r g \frac{\partial v}{\partial x} = r \frac{\partial^2 u}{\partial t^2} \quad \dots(1)$$

$$\frac{\partial s_{12}}{\partial x} + \frac{\partial s_{22}}{\partial x} - p \frac{\partial w}{\partial x} + r g \frac{\partial u}{\partial x} + m_e H_0^2 \left( \frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y} \right) = r \frac{\partial^2 v}{\partial t^2} \quad \dots(2)$$

Where  $\omega$ , is the rotational component about z-axis,  $g$  is the acceleration due to gravity. Where  $\mu_e$  is the magnetic permeability and  $H_0$ , the intensity of the uniform magnetic field, parallel to x-axis, also  $s_{ij}$  is incremental stresses. The incremental stress-strain relation for an incompressible may be taken as,

$$s_{11} = 2N e_{xx} + r, \quad s_{22} = 2N e_{yy} + r \quad \text{and} \quad s_{12} = 2Q e_{xy} \quad \dots(3)$$

$$s_{33} = s_{13} = s_{23}, \quad \text{since the problem is treated in x-y plane where}$$

$r = \frac{(s_{11} + s_{22})}{2}$ ,  $e_{ij}$  is an incremental strain component, and  $N$  and  $Q$  are rigidities of the medium,  $\rho$  a present the density of the medium

$$u = -\frac{\partial j}{\partial y}, \quad v = \frac{\partial j}{\partial x} \quad \dots(4)$$

Substituting from equation (3) and (4) in equations (1) and (2) we get

$$\frac{\partial r}{\partial x} - 2N \frac{\partial^3 j}{\partial x^2 \partial y} + \frac{\partial}{\partial y} \left[ Q \left( \frac{\partial^2 j}{\partial x^2} - \frac{\partial^2 j}{\partial y^2} \right) \right] - \frac{p}{2} \left( \frac{\partial^3 j}{\partial x^2 \partial y} + \frac{\partial^3 j}{\partial y^3} \right) - r g \frac{\partial^2 j}{\partial x} = -r \frac{\partial^3 j}{\partial t^2 \partial y} \quad \dots(5)$$

$$\frac{\partial r}{\partial y} + Q \left( \frac{\partial^3 j}{\partial x^3} - \frac{\partial^3 j}{\partial x \partial y} \right) + \frac{\partial}{\partial y} \left( 2N \frac{\partial^2 j}{\partial x \partial y} \right) - \frac{p}{2} \left( \frac{\partial^3 j}{\partial x^3} + \frac{\partial^3 j}{\partial x \partial y^2} \right) - r g \frac{\partial^2 j}{\partial x \partial y} + m_e H_0^2 \left( \frac{\partial^3 j}{\partial x^3} + \frac{\partial^3 j}{\partial x \partial y^2} \right) = r \left( \frac{\partial^3 j}{\partial t^2 \partial x} \right) \quad \dots(6)$$

Assuming non-homogeneous as

$$\left. \begin{aligned} Q &= Q_0 (1 + a_1 y) \\ N &= N_0 (1 + a_2 y) \\ \rho &= \rho_0 (1 + a_3 y) \end{aligned} \right\} \quad \dots(7)$$

Where  $N_0$  and  $Q_0$  are rigidities and  $\rho_0$  is the density in homogeneous isotropic medium substituting from equation (7) in equation (5) and (6) we get

$$\frac{\partial^2 r}{\partial x \partial y} - [2N_0 a_2 - 2a_1 Q_0 + r_0 g (1 + a_3 y)] \frac{\partial^3 j}{\partial x^2 \partial y} - \left[ \begin{aligned} &2N_0 (1 + a_2 y) \\ &- Q_0 (1 + a_1 y) + \frac{p}{2} \end{aligned} \right] \frac{\partial^4 j}{\partial x^2 \partial y^2} - 2a_1 Q_0 \frac{\partial^3 j}{\partial x^3} - \left[ Q_0 (1 + a_1 y) + \frac{p}{2} \right] \frac{\partial^4 j}{\partial y^4} - r_0 a_3 g \frac{\partial^2 j}{\partial x^2} = -r_0 (1 + a_3 y) \frac{\partial^4 j}{\partial t^2 \partial y} - r_0 a_3 \frac{\partial^3 j}{\partial t^2 \partial y} \quad \dots(8)$$

$$\frac{\partial^3 r}{\partial x \partial y} + \left[ Q_0 (1 + a_1 y) - \frac{p}{2} + m_e H_0^2 \right] \frac{\partial^4 j}{\partial x^4} + \left[ 2N_0 a_2 - r_0 g (1 + a_3 y) \right] \frac{\partial^3 j}{\partial x^2 \partial y} + \left[ 2N_0 (1 + a_2 y) - Q_0 (1 + a_1 y) - \frac{p}{2} + m_e H_0^2 \right] \frac{\partial^4 j}{\partial x^2 \partial y^2} = r_0 (1 + a_3 y) \frac{\partial^4 j}{\partial t^2 \partial x^2} \quad \dots(9)$$

Eliminating  $r$  from equation (8) and (9), we get

$$\begin{aligned} & \left[ Q_0(1+a_1y) - \frac{p}{2} + m_e H_0^2 \right] \frac{\partial^4 j}{\partial x^2} + \left[ 4N_0(1+a_2y) - 2Q_0(1+a_1y) + m_e H_0^2 \right] \frac{\partial^4 j}{\partial x^2 \partial y^2} \\ & + \left[ Q_0(1+a_1y) + \frac{p}{2} \right] \frac{\partial^4 j}{\partial y^4} + 2a_1 Q_0 \frac{\partial^3 j}{\partial y^3} - [2a_1 Q_0 - 4N_0 a_2] \frac{\partial^3 j}{\partial x^2 \partial y} \\ & + r_0 a_3 g \frac{\partial^2 j}{\partial x^2} = r_0(1+a_3y) \left[ \frac{\partial^4 j}{\partial t^2 \partial y^2} + \frac{\partial^4 j}{\partial t^2 \partial x^2} \right] + r_0 a_3 \frac{\partial^3 j}{\partial t^2 \partial y} \quad \dots(10) \end{aligned}$$

### 3. Solution of the Problem

For propagation of sinusoidal waves in any arbitrary direction, we take solution of equation (10) as

$$\varphi(x, y, t) = A e^{ik(xp_1 + yp_2 + dt)} \quad \dots(11)$$

Where  $p_1$  and  $p_2$  are cosine of the angles made by the direction of propagation with x- and y- respectively,  $d$  and  $k$  are the velocity of propagation and wave number respectively

Using equation (11) in equation (10) and equation real and imaginary part separately, we get,

#### Real Part

$$\begin{aligned} \left( \frac{d}{b} \right)^2 = \frac{1}{1+a_3y} & \left\{ (1+a_1y) - \frac{p}{2Q_0} + \frac{m_e H_0^2}{Q_0} \right\} p_1^4 + \left[ \frac{4N_0}{Q_0} (1+a_2y) - 2(1+a_1y) + \frac{m_e H_0^2}{Q_0} \right] \\ & p_1^2 p_2^2 + \left[ (1+a_1y) + \frac{p}{2Q_0} \right] p_2^4 - \frac{a_3 g}{b^2 k^2} p_1^2 \quad \dots(12) \end{aligned}$$

#### Imaginary Part

$$\left( \frac{d}{b} \right)^2 = 2 \frac{a_1}{a_3} p_2^2 + \left[ 4 \frac{N_0}{Q_0} \frac{a_2}{a_3} - \frac{2a_1}{a_3} \right] p_1^2 \quad \dots(13)$$

### 4. Analysis of Problem in Homogeneous Medium

(i) Analysis of equation (12) obtained by equating the real part of equation of motion :

**Case (1):** In this case  $Q$  is homogeneous ( $a_1 \rightarrow 0$ ) i.e. rigidity along vertical direction is constant

$$\begin{aligned} \left( \frac{d}{b} \right)^2 = \frac{1}{1+a_3y} & \left\{ \left[ 1 - \frac{p}{2Q_0} + \frac{m_e H_0^2}{Q_0} \right] p_1^4 \right. \\ & \left. + \left[ -2 + 4 \frac{N_0}{Q_0} (1+a_2y) + \frac{m_e H_0^2}{Q_0^2} \right] p_1^2 p_2^2 + \left[ 1 + \frac{p}{2Q_0} \right] p_2^4 - \frac{a_3 g}{b^2 k^2} p_1^2 \right\} \dots(14) \end{aligned}$$

The velocity along x- direction ( $p_1=1, p_2=0, d=d_{11}$ ) as

$$\left( \frac{d_{11}}{b} \right)^2 = \frac{1}{1+a_3y} \left\{ \left[ 1 - \frac{p}{2Q_0} + \frac{m_e H_0^2}{Q_0} \right] - \frac{a_3 g}{b^2 k^2} \right\} \quad \dots(15)$$

$$d_{11}^2 = \frac{1}{1+a_3y} \left\{ b^2 \left[ 1 - \frac{p}{2Q_0} + \frac{m_e H_0^2}{Q_0} \right] - \frac{a_3 g}{k^2} \right\} \quad \dots(16)$$

Which depends on the initial stress, gravity field and magnetic field. Similarly the velocity of propagation along y- direction ( $p_1=0, p_2=1, d= d_{22}$ ) is obtained as

$$d_{22}^2 = \frac{b^2}{1+a_3y} \left[ 1 + \frac{p}{2Q_0} \right] \quad \dots(17)$$

**Case (2):** in this case N is homogeneous ( $a_2 \rightarrow 0$ ) i.e. rigidity along horizontal direction is constant from equation (12)

$$\left(\frac{d}{b}\right)^2 = \frac{1}{1+a_3y} \left\{ \left[ (1+a_1y) - \frac{p}{2Q_0} + \frac{m_e H_0^2}{Q_0} \right] p_1^4 + \left[ 4 \frac{N_0}{Q_0} - 2(1+a_1y) + \frac{m_e H_0^2}{Q_0} \right] p_1^2 p_2^2 + \left[ (1+a_1y) + \frac{p}{2Q_0} \right] p_2^4 - \frac{a_3 g}{k^2 b^2} p_1^2 \right\} \quad \dots(18)$$

The velocity along x- direction ( $p_1=1, p_2=0, d=d_{11}$ ) is given by

$$d_{11}^2 = \frac{1}{1+a_3y} \left\{ b^2 \left[ (1+a_1y) - \frac{p}{2Q_0} + \frac{m_e H_0^2}{Q_0} \right] - \frac{a_3 g}{k^2} \right\} \quad \dots(19)$$

Which depends on path y, gravity field and magnetic field. The velocity of propagation along y- direction ( $p_1=0, p_2=1, d= d_{22}$ ) is given by

$$d_{22}^2 = \frac{b^2}{1+a_3y} \left[ (1+a_1y) + \frac{p}{2Q_0} \right] \quad \dots(20)$$

For  $p>0$ , the velocity along y- direction may increases considerably at a distance from free surface and the wave because dispersive.

**Case (3):** in this case N, Q and  $\rho$  are homogeneous ( $a_1 \rightarrow 0, a_2 \rightarrow 0, a_3 \rightarrow 0$ )

$$\left(\frac{d}{b}\right)^2 = \left\{ \left[ 1 - \frac{p}{2Q_0} + \frac{m_e H_0^2}{Q_0} \right] p_1^4 + \left[ 4 \frac{N_0}{Q_0} - 2 + \frac{m_e H_0^2}{Q_0} \right] p_1^2 p_2^2 + \left[ 1 + \frac{p}{2Q_0} \right] p_2^4 \right\} \quad \dots(21)$$

In this absence of initial stress the velocity equation becomes (when  $p=0$ )

$$\left(\frac{d}{b}\right)^2 = 1 + \frac{m_e H_0^2}{Q_0} p_1^4 + \left\{ 4 \left[ \frac{N_0}{Q_0} - 1 \right] + \frac{m_e H_0^2}{Q_0} \right\} p_1^2 p_2^2 \quad \dots(22)$$

In x- direction ( $p_1=1, p_2=0, d= d_{11}$ ) the velocity is given by

$$d_{11}^2 = \left[ 1 + \frac{m_e H_0^2}{Q_0} \right] b^2 \quad \dots(23)$$

Any in y- direction ( $p_1=0, p_2=1, d= d_{22}$ ) the velocity is given by

$$d_{22}^2 = b^2$$

**Case (4):** in this case the magnetic field is neglected ( $H_0 \rightarrow 0$ ) the velocity equation is given by

$$\left(\frac{d}{b}\right)^2 = \frac{1}{1+a_3y} \left\{ \left[ (1+a_1y) - \frac{p}{2Q_0} \right] p_1^4 + \left[ 4 \frac{N_0}{Q_0} - 2(1+a_1y) \right] p_1^2 p_2^2 + \left[ (1+a_1y) + \frac{p}{2Q_0} \right] p_2^4 - \frac{a_3 g}{k^2 b^2} p_2^2 \right\} \quad \dots(24)$$

The velocity of a long x- direction ( $p_1=1, p_2=0, d=d_{11}$ ) is given by

$$\left(\frac{d_{11}}{b}\right)^2 = \frac{1}{1+a_3y} \left[ (1+a_1y) - \frac{p}{2Q_0} - \frac{a_3 g}{k^2 b^2} \right] \quad \dots(25)$$

The velocity of a long y- direction ( $p_1=0, p_2=1, d=d_{22}$ ) is given by

$$\left(\frac{d_{22}}{b}\right)^2 = \frac{1}{1+a_3y} \left\{ \left[ (1+a_1y) - \frac{p}{2Q_0} \right] \right\} \quad \dots(26)$$

(ii) Analysis of equation (13) obtained by equating imaginary part of equation of motion in absence the initial stress  $P$  in equation (13) following three cases have been analyzed

**Case(1) :** in this case  $Q$  is homogeneous ( $a_1 \rightarrow 0$ ) i.e. rigidity along vertical direction is constant

$$\left(\frac{d}{b}\right)^2 = \left(\frac{4N_0 a_2}{Q_0 a_3}\right) p_1^2 \quad \dots(27)$$

This show that velocity of shear wave is always damped the velocity of wave along x- direction ( $p_1=1, p_2=0, d=d_{11}$ ) is obtained as

$$\left(\frac{d_{11}}{b}\right)^2 = \left(4 \frac{N_0 a_2}{Q_0 a_3}\right) \quad \dots(28)$$

This show that actual velocity in x- direction is damped by  $(4N_0 a_2 / Q_0 a_3)$  and no damping takes place along y- direction.

**Case (2):** in this case  $N$  is homogeneous ( $a_2 \rightarrow 0$ ) i.e. rigidity along horizontal direction constant

$$\left(\frac{d}{b}\right)^2 = \left(-2 \frac{a_1}{a_3}\right) p_1^2 + 2 \frac{a_1}{a_3} p_2^2 \quad \dots(29)$$

The velocity of wave along x- direction ( $p_1=1, p_2=0, d=d_{11}$ ) is given by

$$\left(\frac{d_{11}}{b}\right)^2 = -2 \frac{a_1}{a_3} \quad \dots(30)$$

The Existence of negative sign shows that damping does not takes place along x- direction for ( $a_2 \rightarrow 0$ ) the velocity along y- direction is given by

$$\left(\frac{d_{22}}{b}\right)^2 = \frac{2a_1}{a_3} \quad \dots(31)$$

Indicating that a damping of magnitude to  $(2a_1/a_3)$  takes place along y- direction.

**Case (3):** in this case  $N$  and  $Q$  are homogeneous ( $a_1 \rightarrow 0, a_2 \rightarrow 0$ ) but density is obtaining higher order approximations in the manner outlined about

$$\left(\frac{d}{b}\right)^2 = 0 \quad \dots(32)$$

i.e., no damping take places. Rather than perform the tedious analysis required in obtaining higher- order approximations in the manner outlined above.

### 5. Numerical Analysis and Discussion

We wish to investigation the variation of shear-waves velocity in a perfectly conducting medium under effect of magnetic field, initial stress and gravity field. To get numerical information on the velocity of shear wave in the non-homogeneous initially stressed medium. We introduce the following non-dimensional parameters:

$$A' = \frac{a_1}{a_2}, B' = a_2 y, C' = \frac{a_3}{a_2}, D' = \frac{d}{a_2}, N' = \frac{N}{Q_0}, P' = \frac{p}{2Q_0}, G' = \frac{g}{a_3^2}, H' = \frac{H}{2Q_0}$$

Using these parameters in the equation (12), we obtain

$$D'^2 = \frac{1}{1+C'B'} \left\{ \begin{aligned} & \left[ (1+A'B') - P' + H' \right] p_1^4 \\ & + [4N'(1+B') - 2(1+A'B') + H'] p_1^2 p_2^2 + \\ & \left[ (1+A'B') + P' \right] p_2^4 - C'G' p_1^2 \end{aligned} \right\} \quad \dots(33)$$

Figures (1-5) show the effect of a non-homogeneous anisotropic incompressible, magnetic field, gravity field and initially stressed respectively on shear waves velocity  $C'$  with respect to depth  $B'$ , it is obvious that shear waves velocity increases with the increasing of the depth  $B'$ , also it is increases with the increasing of the gravity field, magnetic field and it decreases with an increase in of the initial stress.

Fig. 1: Shows the variation in velocities of shear wave in the direction of  $\theta=30^\circ$  with x-axis at different depth and different values of density parameter.

$C'=0.7,0.8,0.9,1.0$ ; taking  $A'=4.0$ ,  $P'=0.5$ ,  $G'=0.1 \text{ cm/sec}^2$ ,  $N'=2.5$  and  $H'=0.3$  The velocity of the shear- wave increases as depth increases and proportional to gravity field.

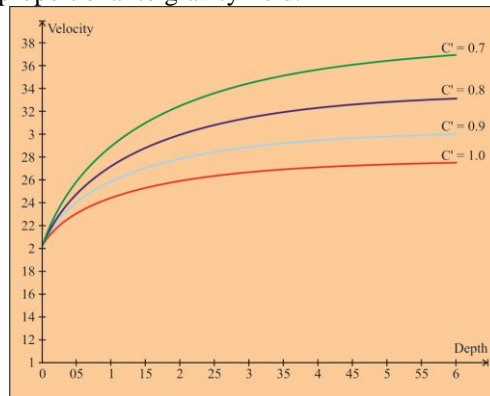


Fig. 2: Shows the variation in velocities of shear wave in the direction of  $\theta=30^\circ$  with x- axis at different depth and different values of magnetic parameters:

$H'=0.1,0.5,0.9,1.3$  taking  $C'=0.8$ ,  $P'=0.5$ ,  $G'=0.1 \text{ cm/sec}^2$ ,  $N'=2.5$  and  $A'=4$  The velocity of the wave increases as depth increases and it is proportional to magnetic field.

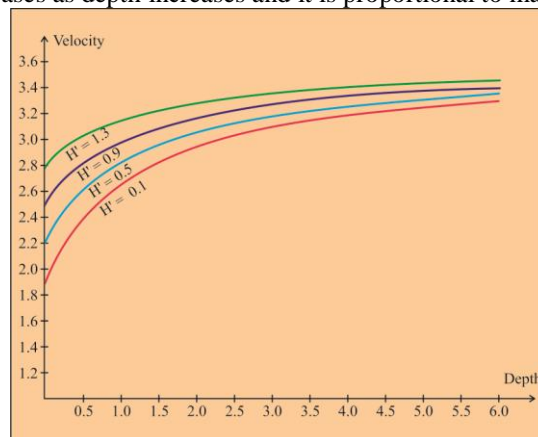


Fig. 3: Shows the variation in velocities of shear-wave in the direction of  $\theta=30^\circ$  with x- axis at different depth and different values of rigidities parameter:

$A'=3.0,3.5,4.0,4.5$  taking  $C'=0.8$ ,  $P'=0.5$ ,  $G'=0.1 \text{ cm/sec}^2$ ,  $N'=2.5$  and  $H'=0.3$  The shear wave velocity increases as depth increases.

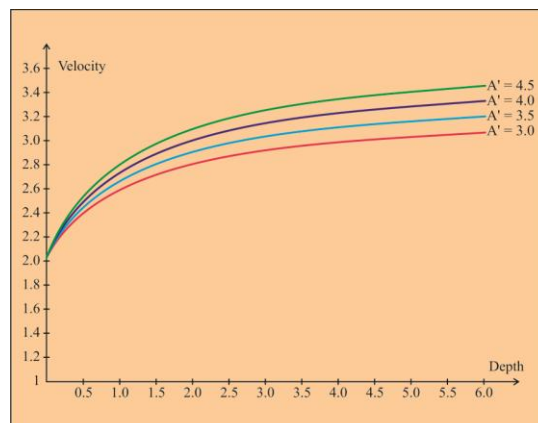


Fig. 4: Shows the variation in velocities of shear wave in the direction of  $\theta=30^\circ$  with x- axis at different depth and different values of  $N'$ :  $N'=2, 2.5, 3.0, 3.5$  taking  $C'=0.8$ ,  $P'=0.5$ ,  $G'=0.1 \text{ cm/sec}^2$ ,  $A'=4$  and  $H'=0.3$ . The shear wave velocity increases as depth increases and proportional to anisotropy.

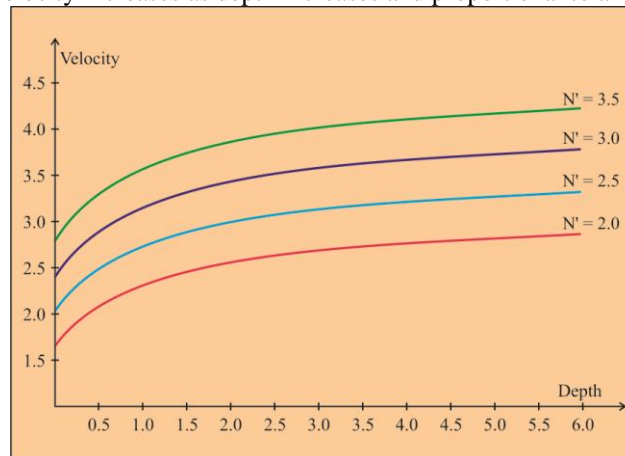
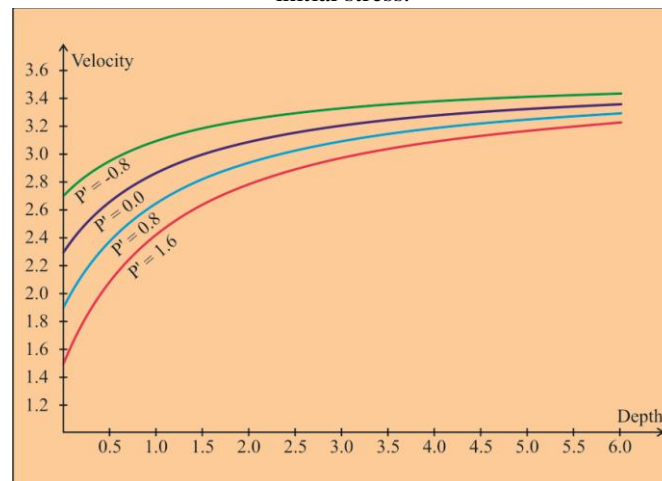


Fig. 5: Shows the variation in velocities of shear wave in the direction of  $\theta=30^\circ$  with x- axis at different depth and different values of initial stress parameter  $P'$ :  $P'=-0.8, 0.0, 0.8, 1.6$  taking  $C'=0.8$ ,  $G'=0.1 \text{ cm/sec}^2$ ,  $A'=4.0$ ,  $N'=2.5$  and  $H'=0.3$ . The velocity of the wave increases as depth increases and it is inversely proportional to initial stress.



## 6. Conclusion

Thus we conclude that the anisotropy, magnetic field, gravity field non-homogeneity of the medium, the initial stress, the direction of propagation and the depth have considerable effect in the velocity of propagation of shear waves and attracts the attention of earth scientists in their work. The presence of initial compressive stress in the medium magnetic field and gravity field, reduce the velocity of propagation while the tensile stress increases it. The shear waves velocities inversely proportional to the initial stress and it is proportional of anisotropy, magnetic field and gravity field.

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