

MHD Heat Transfer of Nanofluids over a stretching sheet with slip effects and chemical reaction

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Abstract: The present study investigates the effect of Chemical Reaction and Magnetic field on heat transfer of a Nanofluid over a Stretching Sheet with Velocity, Thermal and Concentration slips at boundary. A similarity transformation is used to reduce the governing momentum, energy and concentration equations into non-linear ordinary differential equations which are solved numerically using the Keller box method. Velocity and temperature profiles as well as the skin friction coefficient, the Nusselt number and Sherwood number are determined numerically. The influence of relevant parameters such as Prandtl number, Brownian motion, Thermophoresis parameters, Lewis number on the flow and heat transfer characteristics are discussed in graphs and tabulated form.

Keywords: MHD, Heat transfer, Nanofluids, velocity slip, Thermal slip, Chemical Reaction

Nomenclature:

List of variables:

u, v : Velocity components in the x - and y - axis, respectively (m/s)

U_w : Velocity of the wall along the x -axis (m/s)

x, y : Cartesian coordinates measured along the stretching sheet (m)

$B(x)$: Magnetic field strength ($A m^{-1}$)

C : Nano particle concentration ($mol m^{-3}$)

C_{fx} : Skin-friction coefficient (*Pascal*)

Nu_x : Nusselt number

Sh_x : Sherwood number

C_w : Nano particles concentration at the stretching surface ($mol m^{-3}$)

C_∞ : Nano particle concentration far from the sheet ($mol m^{-3}$)

C_p : Specific heat capacity at constant pressure ($J Kg^{-1} K$)

D_T : Brownian diffusion coefficient

D_B : Thermophoresis diffusion coefficient

a : Constant parameter

R : Thermal radiation parameter

S : Suction/Injection parameter

f : Dimensionless stream function

Le : Lewis number

M : Magnetic parameter

Nb : Brownian motion parameter

Nt : Thermophoresis parameter

Pr : Prandtl number

Re_x : Reynolds number

T : Fluid temperature (K)

T_w : Temperature at the surface (K)

T_∞ : Temperature of the fluid far away from the stretching sheet

q_w : Surface heat flux (W/m^2)

q_m : Surface mass flux

Greek Symbols:

α : Thermal diffusivity (m^2/s)

ψ : Stream function

η : Dimensionless similarity variable

μ : Dynamic viscosity of the base fluid ($kg/m.s$)

ν : Kinematic viscosity ($m^2 s^{-1}$)

ρ_f : Density of the fluid ($Kg m^{-3}$)

ρ_p : Density of the nanoparticle ($Kg m^{-3}$)

τ : The ratio of the nanoparticle heat capacity and the base fluid heat capacity

$(\rho c)_f$: Heat capacity of the base fluid ($kg/m.s^2$)

$(\rho c)_p$: Heat capacity of the nano particle ($kg/m.s^2$)

θ : Dimensionless temperature (K)

p : Pressure (N/m^2)

ϕ : Nanoparticle volume fraction

ϕ_w : Nanoparticle volume fraction at wall temperature

ϕ_∞ : Ambient nanoparticle volume fraction

λ : Velocity slip parameter

δ : Thermal slip parameter

γ : Solutal slip parameter

K : Chemical reaction parameter

Q : Heat source parameter

Sub Scripts:

f : Fluid

W : Condition on the sheet

∞ : Ambient Conditions

Superscripts:

' : Differentiation w.r.t η

1. Introduction

The study of boundary layer flow and heat transfer due to stretching surface has become more and more important in many engineering process with industrial applications, such as in polymer extrusion, drawing of copper wires, artificial fibers, paper production, hot rolling, wire drawing, glass fiber, metal extrusion and metal spinning, and continuous stretching of plastic films. P.S.Gupta and A.S.Gupta [1] examined that heat and mass transfer on a stretching sheet with suction or blowing. . Numerical study of entropy generation for forced convection flow and heat transfer of a Jeffrey fluid over a stretching sheet was investigated by Nemat Dalir [2]. Exact analytical solutions for the flow and heat transfer near the stagnation point on a stretching/shrinking sheet in a Jeffrey fluid were studied by M. Turkyilmazoglu and I. Pop[3]. Heat transfer in a viscoelastic boundary layer flow over a stretching sheet with viscous dissipation and non-uniform heat source derived by M. Subhas Abel et.al [4]. Liancun Zheng et.al.[5] studied analytic solutions of unsteady boundary flow and heat transfer on a permeable stretching sheet with non-uniform heat source/sink.

Magnetohydrodynamics (MHD) is concerning the mathematical and physical scaffold that introduces magnetic dynamics in electrically conducting fluids. MHD applications are used in modern industrial and engineering fields like in the extrusion of a polymer in a melt-spinning process, drawing of plastic films and wires, manufacture of foods, crystal growing, glass fiber and paper production, liquid film in condensation process, electronic chips, thermal energy storage, electrochemical process, flow through filtering devices and many of others. MHD flow of a dusty fluid near the stagnation point over a permeable stretching sheet with non-uniform source/sink were derived by G.K. Ramesh et.al[6]. MHD stagnation-point flow and heat transfer towards stretching sheet with induced magnetic field was introduced by F. M. Ali et.al[7]. Dual solutions in MHD stagnation-point flow of Prandtl fluid impinging on shrinking sheet was derived by N. S. Akbar et.al[8]. Behrouz Raftari and K. Vajravelu[9] analyzed Homotopy analysis method for MHD viscoelastic fluid flow and heat transfer in a channel with a stretching wall.

Nanofluids are basically in the form of dispersing solid nanometer-sized particles in fluids such as water, oil or ethylene glycol. The nanoparticles used in nanofluids are typically made of metals (Cu, Ag), oxides (Al₂O₃), carbides (SiC), nitrides (AlN, SiN) or nonmetals (Graphite, carbon nanotubes) and the base fluid is usually a conductive fluid. Nanotechnology has been widely used in industry since materials with size of nanometers possess unique physical and chemical properties. The term “nanofluid” was first introduced by Choi[10]. Flow and heat transfer of a nanofluid over a nonlinearly stretching sheet: A numerical study was derived by P. Rana and R. Bhargava[11]. MHD stagnation point flow and heat transfer due to nanofluid towards a stretching sheet was introduced by Ibrahim et.al[12]. Unsteady boundary-layer flow and heat transfer of a nanofluid over a permeable stretching/shrinking sheet was derived by Norfifah Bachok et.al[13]. Convective heat transfer in the flow of viscous Ag–water and Cu–water nanofluids over a stretching surface was analyzed by K. Vajravelu et.al[14]. Navid Freidoonimehr et.al[15] introduced unsteady MHD free convective flow past a permeable stretching vertical surface in a nanofluid. Entropy generation analysis of magneto hydrodynamic flow of a nanofluid over a stretching sheet was investigated by M. Govindaraju et.al[16]. Magnetic field effect on nanofluid flow and heat transfer using KKL model was analyzed by M. Sheikholeslami et.al[17]. Multiple solutions of MHD boundary layer flow and heat transfer behavior of nanofluids induced by a power-law stretching/shrinking permeable sheet with viscous dissipation was derived by Ruchika Dhanai et.al[18]. Boundary layer flow of Heat transfer and Nanofluids over a nonlinear stretching sheet with presence of magnetic field and viscous dissipation was investigated by shravani et.al.[19].

The effects of radiation on MHD nanofluid flow are significant in several processes occurring at high temperature in engineering areas and so knowledge of radiation heat transfer becomes very significant exclusively for manufacturing of reliable equipment, nuclear plants, gas turbines, satellites, various propulsion devices for aircraft, missiles and space vehicles, etc. Krishnendu Bhattacharyya and G.C. Layek[20] introduced Effects of suction/blowing on steady boundary layer stagnation-point flow and heat transfer towards a shrinking sheet with thermal radiation. MHD boundary layer flow and heat transfer over an exponentially stretching sheet embedded in a thermally stratified medium was derived by Swati Mukhopadhyay[21]. Buoyancy effect on MHD flow of nanofluid over a stretching sheet in the presence of thermal radiation was derived by M.M. Rashidi et.al[22]. Effects of thermal radiation on micropolar fluid flow and heat transfer over a porous shrinking sheet was analyzed by Krishnendu Bhattacharyya et.al[23]. Mahantesh M. Nandeppanavar et.al[24] derived Heat transfer in MHD viscoelastic boundary layer flow over a stretching sheet with thermal radiation and non-uniform heat source/sink.S. Pramanik[25] analyzed Casson fluid flow and heat transfer past an exponentially porous stretching surface in presence of thermal radiation.

The study of chemical reaction with heat transfer in porous medium has important engineering applications e.g. tubular reactors, oxidation of solid materials and synthesis of ceramic materials. The combined effects of heat and mass transfer with chemical reaction are of great importance to engineers and scientists

because of its almost universal occurrence in many branches of science and engineering, and hence received a considerable amount of attention in recent years. Diffusion of chemically reactive species in third grade fluid flow over an exponentially stretching sheet considering magnetic field effects was derived by T. Hayat et.al[26]. MHD stagnation point flow and heat transfer impinging on stretching sheet with chemical reaction and transpiration was investigated by F.Mabood et.al[27]. T. Hayat et.al[28] analyzed Impacts of constructive and destructive chemical reactions in magnetohydrodynamic (MHD) flow of Jeffrey liquid due to nonlinear radially stretched surface. Thermophoresis and Brownian motion effects on MHD bioconvection of nanofluid with nonlinear thermal radiation and quartic chemical reaction past an upper horizontal surface of a paraboloid of revolution was introduced by O.D.Makinde and .I.L.Animasaun[29]. Chemical reaction effect on MHD boundary-layer flow of two-phase nanofluid model over an exponentially stretching sheet with a heat generation was introduced by Mohamed R. Eid[30]. Influence of chemical reaction and heat source on dissipative MHD mixed convection flow of a Casson nanofluid over a nonlinear permeable stretching sheet was derived by S.M. Ibrahim et.al[31]. T. Hayat et.al.[32] analyzed mixed convection flow of viscoelastic nanofluid by a cylinder with variable thermal conductivity and heat source/sink. Effect of partial slip on hydromagnetic flow over a porous stretching sheet with non-uniform heat source/sink, thermal radiation and wall mass transfer was introduced by A.K.Abdul Hakeem et.al.[33]. Soret and Dufour effects in three-dimensional flow over an exponentially stretching surface with porous medium, chemical reaction and heat source/sink was investigated by T. Hayat et.al.[34]

Slip Effects on an Unsteady Boundary Layer Stagnation-Point Flow and Heat Transfer towards a Stretching Sheet was derived by Krishnendu Bhattacharyya et.al[35]. MHD boundary layer flow and heat transfer of a nanofluid past a permeable stretching sheet with velocity, thermal and solutal slip boundary conditions was investigated by Wubshet Ibrahim and Bandari Shankar[36]. Kai-Long Hsiao[37] analyzed Stagnation electrical MHD nanofluid mixed convection with slip boundary on a stretching sheet. Thermal radiation and slip effects on MHD stagnation point flow of nanofluid over a stretching sheet was introduced by Rizwan Ul Haq et.al[38]. Slip effects on MHD boundary layer flow over an exponentially stretching sheet with suction/blowing and thermal radiation was studied by Swati Mukhopadhyay[39].

The present study is a generalized approach by which we can discuss the results of Pal and Roy[40] and we add specific parameters Magnetic, chemical reaction and slips. The present study is to examine the simultaneous effects of thermal jump, solutal slip boundary conditions and magnetic parameters on flow, heat transfer characteristics over a stretching sheet in the presence of chemical reaction. The governing equations are solved numerically using Keller-box method.

2. Mathematical Formulation:

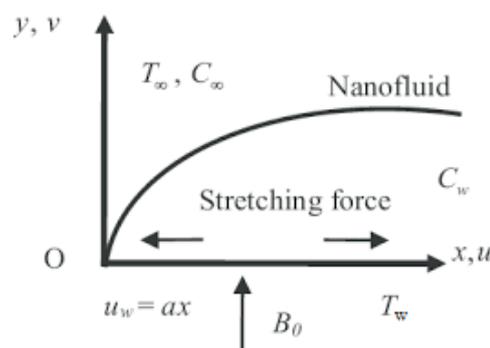


Figure 1. Physical model and coordinate system.

Consider a two-dimensional steady state laminar boundary layer flow of a nanofluid over stretching sheet with surface temperature T_w and concentration C_w . The stretching velocity of the sheet is $u_w = ax$, where a is a constant. The ambient temperature and concentration, respectively, are T_∞ and C_∞ . The induced magnetic field is also assumed to be small compared to the applied magnetic field; so it is neglected. It is further assumed that the base fluid and the suspended nanoparticles are in thermal equilibrium. In Cartesian coordinates system of x and y , the governing equations of conservation of momentum, thermal energy and nanoparticles volume fraction under boundary layer approximation including the dynamic effects of nanoparticles can be written as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2(x)}{\rho_f} u, \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_m \nabla^2 T + \tau \left\{ D_B \frac{\partial \phi}{\partial y} \frac{\partial T}{\partial x} + \frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 \right\} - \frac{1}{(\rho c_p)_f} \frac{\partial q_r}{\partial y} + Q_0 \frac{(T-T_\infty)}{(\rho c_p)_f} \tag{3}$$

$$u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} = D_B \frac{\partial^2 \phi}{\partial y^2} + \left(\frac{D_T}{T_\infty} \right) \frac{\partial^2 T}{\partial y^2} - K_r (C - C_\infty) \tag{4}$$

Here u and v are the velocity components along the x and y directions, respectively, ρ_f the density of the base fluid, α_m the thermal diffusivity, ν the kinematic viscosity, D_B the Brownian diffusion coefficient, D_T the thermophoretic diffusion coefficient,

$\tau = \frac{(\rho c)_p}{(\rho c)_f}$ the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid,

c is the volumetric volume expansion coefficient, and ρ the density of the particles.

Using Rosseland approximation for radiation we can write

$$q_r = -\frac{4\sigma^* \partial T^4}{3k^* \partial y} \tag{5}$$

Where σ^* is the Stefan–Boltzman constant, k^* is the absorption coefficient. Assuming that the temperature difference within the flow is such that T^4 may be expanded in a Taylor series and expanding T^4 about T_∞ , the free stream temperature and neglecting higher orders we get $T^4 \equiv 4T_\infty^3 T - 3T_\infty^4$.

We introduce subjective boundary conditions are

$$u = U_w(x) + A \frac{\partial u}{\partial y}, v = v_0, T = T_w + B \frac{\partial T}{\partial y}, \phi = \phi_w + C \frac{\partial \phi}{\partial y} \text{ as } y = 0$$

$$u = 0, v = 0, T = T_\infty, \theta = \theta_\infty, \phi = \phi_\infty \text{ as } y \rightarrow \infty \tag{6}$$

Introducing similarity transformations

$$\psi = (av)^{1/2} x f(\eta), \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \phi(\eta) = \frac{C-C_\infty}{C_w-C_\infty}, \eta = \left(\frac{a}{v}\right)^{1/2} y \tag{7}$$

Where the stream function $\psi(x, y)$ is defined as:

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \tag{8}$$

Using the variables defined by eq. (7), the momentum equation, eq. (2), the energy equation, eq. (3), and the equation for the solid volume fraction of nanofluid, eq. (4) may be rewritten as:

$$f''' + ff'' - f'^2 - Mf' = 0 \tag{9}$$

$$\left(1 + \frac{4}{3R}\right) \theta'' + Pr(f\theta' + Nb\phi'\theta' + Nt\theta'^2 + Q\theta) = 0 \tag{10}$$

$$\phi'' + Le f\phi' + \frac{Nt}{Nb} \theta'' - LeK\phi = 0 \tag{11}$$

with the transformed boundary condition

$$f(0) = S, f'(0) = 1 + \lambda f''(0), \theta(0) = 1 + \delta \theta'(0), \phi(0) = 1 + \gamma \phi'(0) \tag{12}$$

$$f'(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0, \phi(\eta) \rightarrow 0 \text{ as } \eta \rightarrow \infty$$

where prime denotes differentiation with respect to η . S is the suction-injection parameter ($S > 0$ for suction, $S < 0$ for injection) and the dimensionless parameters Nb, Nt, Pr, Le, M, S, R, λ , δ , γ , K, Q are the Brownian motion parameter, the thermophoresis parameter, Prandtl, Lewis, the magnetic parameter, the suction-injection parameter, Thermal radiation parameter, velocity, thermal, solutal slip parameters, chemical reaction parameter, heat source parameter respectively. These parameters are defined as follows:

$$Nb = \frac{\rho c_p D_B (\phi_w - \phi_\infty)}{\rho c_f \nu}, Nt = \frac{\rho c_p D_T (T_w - T_\infty)}{\rho c_f \nu T_\infty}, Pr = \frac{\nu}{\alpha}, Le = \frac{\nu}{D_B}, M = \frac{\sigma B_0^2 x}{\rho u_\infty} \tag{13}$$

$$S = -v_w(x) \sqrt{\frac{2x}{\nu u_\infty}}, R = \frac{KK^*}{4\sigma^* T_\infty^3}, \lambda = A \sqrt{\frac{a}{\nu}}, \delta = B \sqrt{\frac{a}{\nu}}, \gamma = C \sqrt{\frac{a}{\nu}}, K = \frac{\nu K_r}{U_w^2},$$

$$Q = \frac{Q_0}{a(\rho c_p)_f}$$

The physical quantities of interest are the skin frictions C_f , the local Nusselt number Nu_x and the Sherwood number Sh_x which are defined as

$$C_f = \frac{2\tau_w}{\rho u_w^2}, Nu_x = \frac{xq_w}{K(T_w - T_\infty)}, Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)} \tag{14}$$

Where τ_w is the surface shear stress, q_w is the surface heat flux and q_m is the surface mass flux.

3. Results and Discussions:

The numerical solutions are obtained for velocity, temperature, concentration profiles, skin friction, Nusselt number and Sherwood number for different values of governing parameters. The obtained results are displayed through graphs. Figs. 2–16 for velocity, temperature and concentration profiles, respectively. To formalize the present solution, comparisons have been made with previously published data in the literature for $-\theta'(0)$ and $-\phi'(0)$ in Table 1. They are found to be in an excellent agreement. Effects of magnetic, chemical reaction, thermal slip and concentration slip parameters on Skin friction factor, Nusselt and Sherwood numbers are presented in Tables 2 and 3, while keeping other parameter values present.

Table.1: Comparison of Nusselt number and Sherwood number ($-\theta'(0)$) and ($-\phi'(0)$) when the values $Pr = Le = 10, \lambda = \alpha = s = 0, R = \infty$.

Nt	Nb	Noghrehabadi[41] (Nu_x)	Pal[40] (Mu_x)	Present results (Nu_x)	Noghrehabadi[41] (Sh_x)	Pal[40] (Sh_x)	Present results (Sh_x)
0.1	0.1	0.9523	0.9523	0.9523	2.1293	2.1293	2.1291
0.2	0.1	0.6931	0.6931	0.6932	2.2740	2.2740	2.2736
0.3	0.1	0.5200	0.5200	0.5201	2.5286	2.5286	2.5280
0.4	0.1	0.4025	0.4025	0.4026	2.7951	2.7951	2.7943
0.5	0.1	0.3210	0.3210	0.3211	3.0351	3.0351	3.0340
0.1	0.2	0.5055	0.5036	0.5055	2.3818	2.3569	2.3817
0.1	0.3	0.2521	0.2518	0.2521	2.4100	2.4017	2.4099
0.1	0.4	0.1194	0.1193	0.1194	2.3996	2.3950	2.3995
0.1	0.5	0.0542	0.0542	0.0542	2.3835	2.3797	2.3834

Table 2:

Calculation of $-f''(0), -\theta'(0)$ and $-\phi'(0)$ when $Nb = Nt = S = \lambda = Ec = Q = \delta = \gamma = 0.1, Le = Pr = 2.0$.

M	K	Skinfriction $f''(0)$	Nusselt Number Nu_x	Sherwood number Sh_x
0.0	0.1	1.2104	0.6970	0.6377
0.3		1.3578	0.6703	0.6209
0.5		1.4475	0.6538	0.6117
1.0		1.6500	0.6165	0.5940
0.1	0.0	1.2614	0.6899	0.5594
	0.3	1.2614	0.6842	0.7618
	0.5	1.2614	0.6812	0.8768
	1.0	1.2614	0.6755	1.1196

Table3

Calculation of $-f''(0), -\theta'(0)$ and $-\phi'(0)$ when $Nb = Nt = S = \lambda = Ec = Q = M = K = 0.1, Le = Pr = 2.0$.

δ	γ	Skinfriction $f''(0)$	Nusselt Number Nu_x	Sherwood number Sh_x
0.0	0.1	1.2614	0.7521	0.5958
0.3			0.5874	0.6881
0.5			0.5125	0.7304
1.0			0.3886	0.8009
0.1	0.0	1.2614	0.6810	0.7401
	0.3		0.6984	0.4648
	0.5		0.7062	0.3424
	1.0		0.7191	0.1433

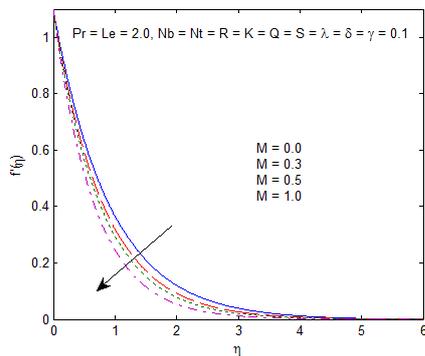


Fig 2. Effect of Magnetic parameter (M) on velocity profile.

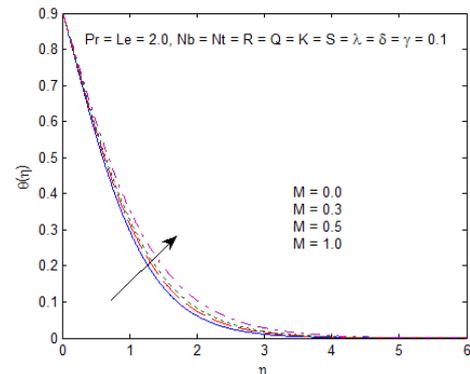


Fig 3. Effect of Magnetic parameter (M) on temperature profile.

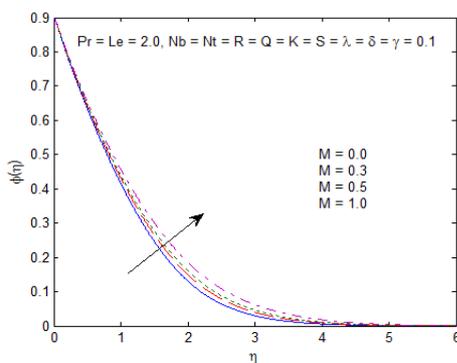


Fig 4. Effect of Magnetic parameter (M) on Concentration profiles.

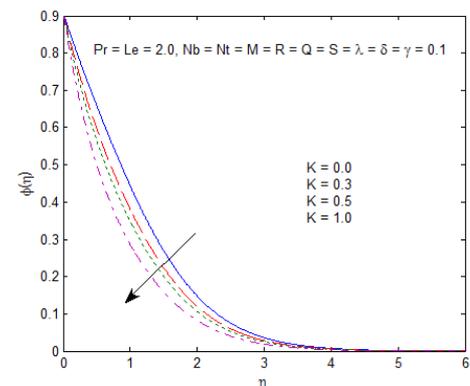


Fig 5. Effect of Chemical reaction parameter (K) on Concentration profiles.

Fig.2 illustrates the effect of magnetic field (M) on the velocity profiles. It is observed that the velocity of the fluid decreases as magnetic field increase. It is due to fact that Lorentz force. It means magnetic field introduces a retarding body force which behaves crosswise to the direction of the applied magnetic field. This body force reduced the boundary layer flow and thickens the momentum boundary layer.

Figs.3 and 4 shows the effect of magnetic field (M) on temperature and concentration profiles. From these figures we can see that both the temperature and the concentration profiles demonstrated an increasing behavior for raising the values of M . Physical significance of this behavior is, As Lorentz force is a frictional resistive force opposes the fluid motion and then it is heat produced. According to result, the thermal boundary layer thickness and concentration boundary layer thickness become thicker for stronger magnetic field.

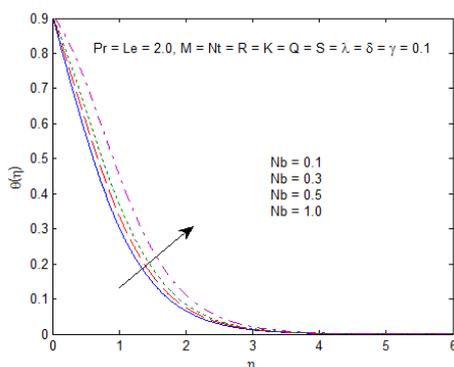


Fig 6. Effect of Brownian motion (Nb) on temperature profile.

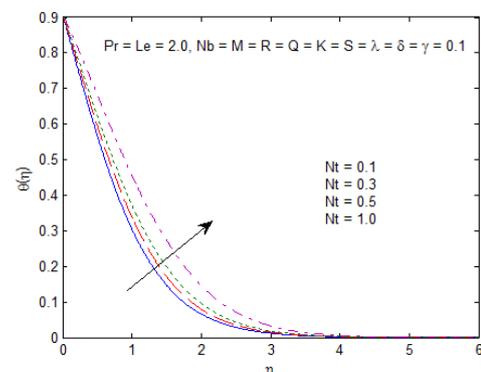


Fig 7: Effect of Thermophoresis (Nt) on temperature profile.

Fig.5 shows the effect of Chemical reaction parameter(K) on concentration profile. From this figure we observed that increasing the value of the chemical reaction decreases the concentration profile in the boundary

layer. The physical significance of this phenomena is negative chemical reduces the concentration boundary layer thickness and increases the mass transfer. Figs 6 and 7 shows the influence of the change of Brownian motion parameter Nb and thermophoresis parameter Nt on temperature profile. It is noticed that as thermophoresis parameter increases the thermal boundary layer thickness increases and the temperature gradient at the surface decrease (in absolute value) as both Nb and Nt increase. This is due to the fact that the temperature gradient generates a thermophoretic force and it creates a fast flow away from the stretching sheet. In this way more heated fluid is traveled away from the surface, and consequently, as Nt increases, the temperature within the boundary layer increases. So that an increase in the Brownian motion parameter Nb thickens the thermal boundary layer.

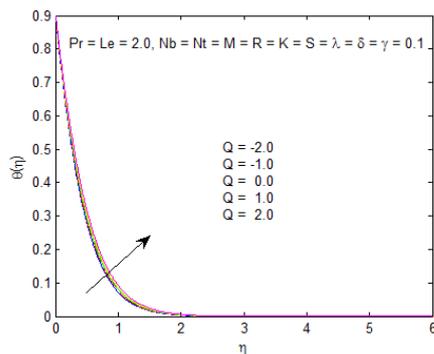


Fig 8: Effect of Heat source parameter (Q) on Temperature profile.

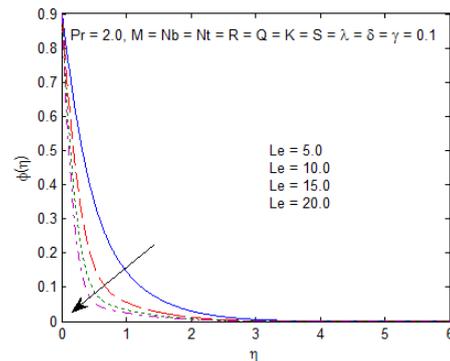


Fig 9: Effect of Lewis number (Le) on concentration profile

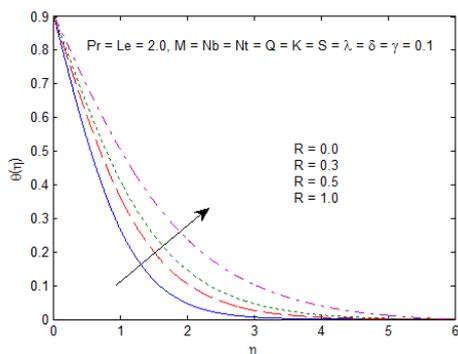


Fig 10: Effect of Thermal Radiation (R) on Temperature profile.

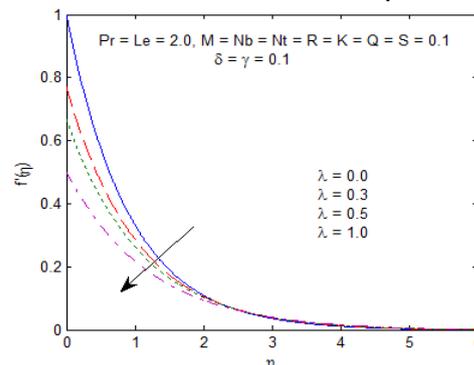


Fig 11: Effect of Velocity Slip Parameter (lambda) on velocity profile.

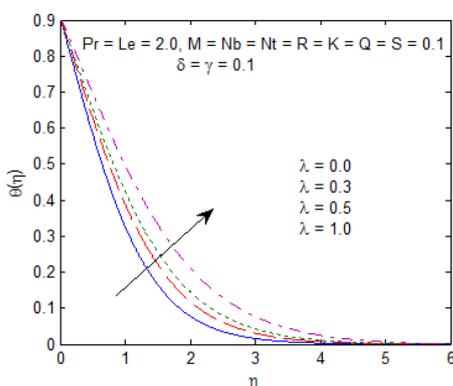


Fig 12: Effect of Velocity Slip Parameter (lambda) on temperature profile.

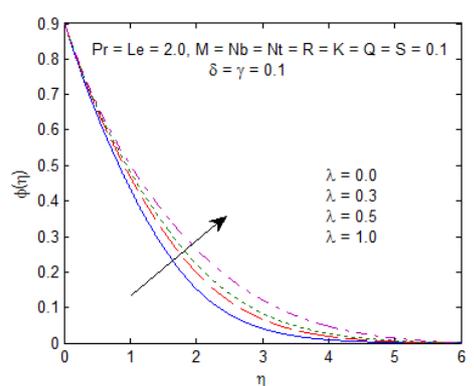


Fig 13: Effect of Velocity Slip Parameter (lambda) on Concentration profile.

The influence of heat generation/ absorption parameter Q on the temperature profile are shown in Fig. 8. Here we observed that when the Heat generation/absorption parameter Q increases then the temperature profile increased. Increasing the heat generation parameter Q has the tendency to increase the thermal state of the fluid.

This increase in the temperature fluid causes more induced flow towards the plate through the thermal buoyancy effect. Fig.9 shows the effect of Lewis number (Le) on the nanoparticle volume fraction. As the Lewis number Le gains then decelerate in the Concentration profiles. The larger values of Lewis number creates the lower molecular diffusivity, therefore it decreases the concentration field. The influence of thermal radiation parameter (R) on the temperature profile are shown in Fig.10. Here we observed that when the radiation parameter increases then the temperature profile increased.

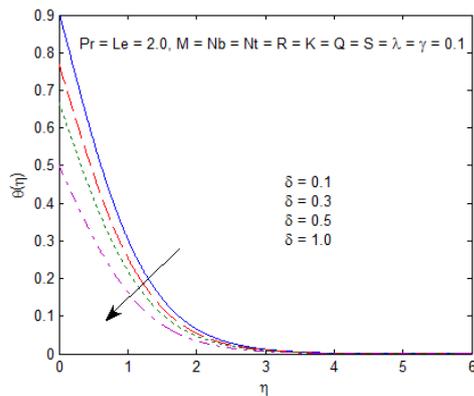


Fig 14: Effect of Thermal Slip Parameter (δ) on temperature profile

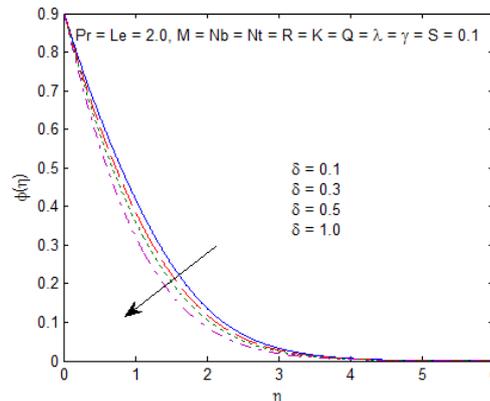


Fig 15: Effect of Thermal Slip Parameter (δ) on concentration profile.

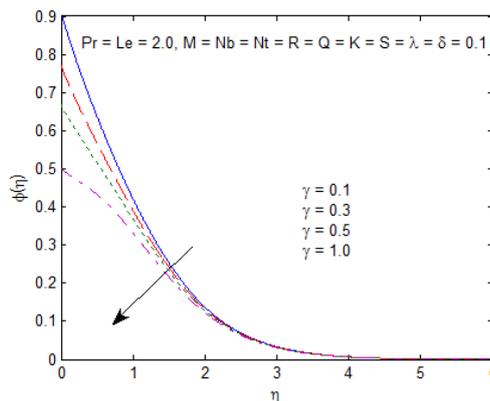


Fig 16: Effect of Concentration Slip Parameter (γ) on concentration profile

Fig. 11 depicts that the effect of velocity slip parameter λ on the velocity profile. As velocity slip parameter λ increases then there is a reduction in velocity profile. It is observed that slip velocity is increases and consequently fluid velocity decreased because of the slip condition at the boundary. The pulling of the stretching sheet can only partly be transmitted to the fluid. It is found that velocity slip λ has a substantial effect on the solutions.

Fig. 12 shows that the effects of velocity slip parameter λ on the temperature profiles. For the variation of velocity slip parameter λ in presence of suction. With the increasing λ , the temperature is found to be decreased initially but after a certain distance from the sheet it increases with slip parameter λ .

Fig. 13 depicts that the effect of velocity slip parameter λ on the concentration profile. As velocity slip parameter λ increases then there is a strong increment in concentration profile. It is observed that slip velocity is increases and consequently concentration fluid is increased because of the slip condition at the boundary.

Fig. 14 shows the effects of thermal slip parameter (δ) on temperature profile. Initially the temperature decreases when thermal slip parameter is increased. Then less heat is transferred to the fluid from the sheet and so temperature is found to be decrease.

Fig.15 depicts that the effect of on the thermal slip parameter (δ) on concentration profile. As thermal slip parameter (δ) increases then there is a reduction in concentration profile. It is observed that thermal slip δ is increases and consequently concentration fluid is decreased. The influence of concentration slip parameter (γ) on the nanoparticle concentration profile are shown in

Fig 16. From this figure it can be observed as concentration slip parameter (γ) increases, then there is a diminution in concentration profile. It is due to the fact that slip basically retards the fluid motion which finally shows a decrease in concentration profile.

4. Conclusions:

A numerical study of the boundary layer laminar flow in a nanofluid over stretching sheet has been performed. A similarity solution is presented which depends on Lewis numbers, magnetic, Brownian motion and the thermophoresis, chemical reactions, heat source, thermal radiation, velocity slip, thermal slip and concentration slip parameters. The effects of governing parameters on the velocity, temperature, concentration flow, heat and mass transfer characteristics are presented graphically and quantitatively. The observations of the present study are as follows:

1. By increasing the values of Magnetic Parameter M and Velocity slip parameter λ on Velocity, Temperature and Concentration profiles then there is reduction in velocity profile and increase in the Temperature and Concentration profiles.
2. When the Thermal Slip Parameter δ increases, then it reduces the temperature and concentration profiles.
3. Temperature Profile is increased with the increasing values of Heat generation/absorption parameter Q .
4. Concentration profile is reduced when the chemical reaction parameter K , Concentration Slip Parameter γ and Lewis number Le are increased.
5. By increasing the values of Brownian motion parameter (Nb) and Thermophoresis parameter (Nt) then the Temperature profiles are increased
6. Temperature Profile is increased with the increasing values of Thermal radiation parameter R .
7. Rate of heat transfer rate decreases, whereas mass transfer rate decreases with Magnetic field.
8. Rate of heat transfer rate decreases, whereas mass transfer rate increases Chemical reaction K
9. Rate of Nusselt number decreases, whereas mass Sherwood number increases with thermal slip δ .
10. Rate of Nusselt number increases, whereas Sherwood number decreases with concentration slip γ .

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