

## A Simple Optimization Model for Minimizing Weight of a Shaft

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**Abstract:** Shaft weight optimization is an analysis to get the optimum value of the shaft dimensions which will produce the minimum weight of the shaft. The dimensions of the shaft are the outer diameter and thickness of the shaft. This research develops a simple model to optimize the weight of the shaft with various driving power values. The constraint functions are stress and displacement on the shaft. In order to evaluate the proposed optimization model, a shaft that has torsion and bending is analyzed. Finally, the optimization results have been obtained, namely optimum outer diameter and shaft thickness values for the driving power of 1 hp to 10 hp. This result can be a guideline for technical information for practitioners and designers in the industry.

**Keywords:** Optimization, model, shaft, weight

### I. INTRODUCTION

Optimizing components or machine elements means an analysis to get the minimum or maximum value of the parameters related to the component or element of the machine. Optimization is often needed when an initial design of a component is analyzed for its performance, then based on the analysis it is found that the component can be improved.

Optimization has been applied to various areas. Messaoud and Abdessamed proposed a control strategy of a variable speed wind generation system. The aim of this control strategy is to allow the permanent magnet generator to operate for different wind speed in order to optimize the generated power from wind turbine on the one hand [1]. Kusiak and Zheng discussed an evolutionary computation approach for optimization of power factor and power output of wind turbines [2]. Eminoglu and Ayasun conducted a study focused on calculation of the power output of variable-speed wind turbine systems [3]. Shen et al. introduced reliability-based design optimization performed considering manufacturing uncertainties in order to facilitate the performance and reliability of a centrifugal compressor [4]. Thamarai Kannan and Thirunavukkarasu studied a teaching-learning based optimization algorithm with differential operator for optimization task of mechanical components [5]. These studies involve quite complex parameters and calculations.

Because the existing optimization studies are still quite complex, the idea is to create a simple optimization model. For example a simple model, that can optimize well the weight of the shaft. The purpose of this study is to create a simple optimization model for minimizing weight of the shaft with variations in driving force. The optimum outer diameter and shaft thickness values will produce optimum cross-sectional area, which will also produce the minimum weight of the shaft.

### II. PROPOSED OPTIMIZATION MODEL

Generally, the optimization problem can be stated as follows [6], minimize or maximize:

$$F(\mathbf{X}) ; \text{ to find } \mathbf{X} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ \vdots \\ \vdots \\ \vdots \\ X_n \end{pmatrix} \dots\dots\dots(1)$$

Where  $F(\mathbf{X})$  is objective function, and  $X_1, X_2, \dots, X_n$  are design variables, subject to the constraints :

$$g_j(\mathbf{X}) \leq 0 \quad , j = 1 \dots m : \text{ inequality constraints } \dots\dots\dots(2)$$

$$h_k(\mathbf{X}) = 0, \quad k = 1 \dots 1 : \text{equality constraints} \dots\dots\dots(3)$$

$$X_i^L \leq X_i \leq X_i^U, \quad i = 1 \dots n : \text{side constraints} \dots\dots\dots(4)$$

Optimization problem also can be illustrated as depicted in Figure 1. This figure explains the relationship among objective functions, design variables and constraint functions.

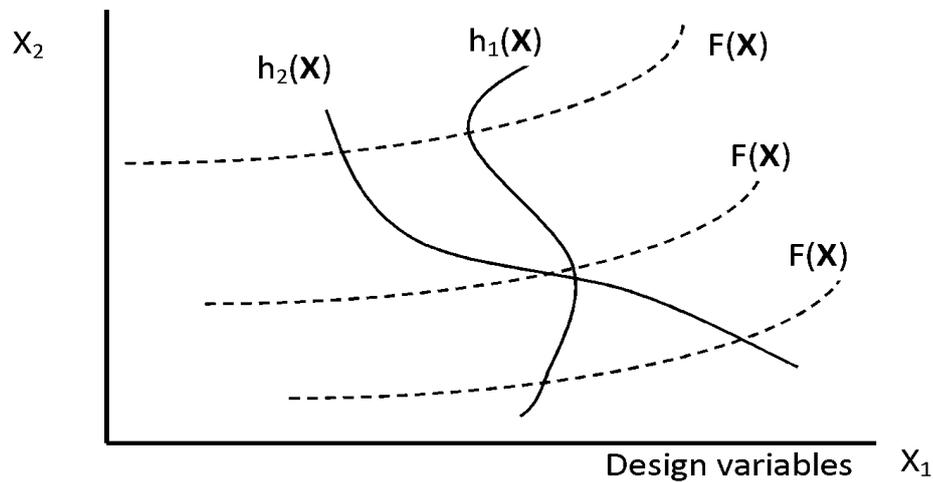


Figure 1. Objective functions, design variables and constraint functions

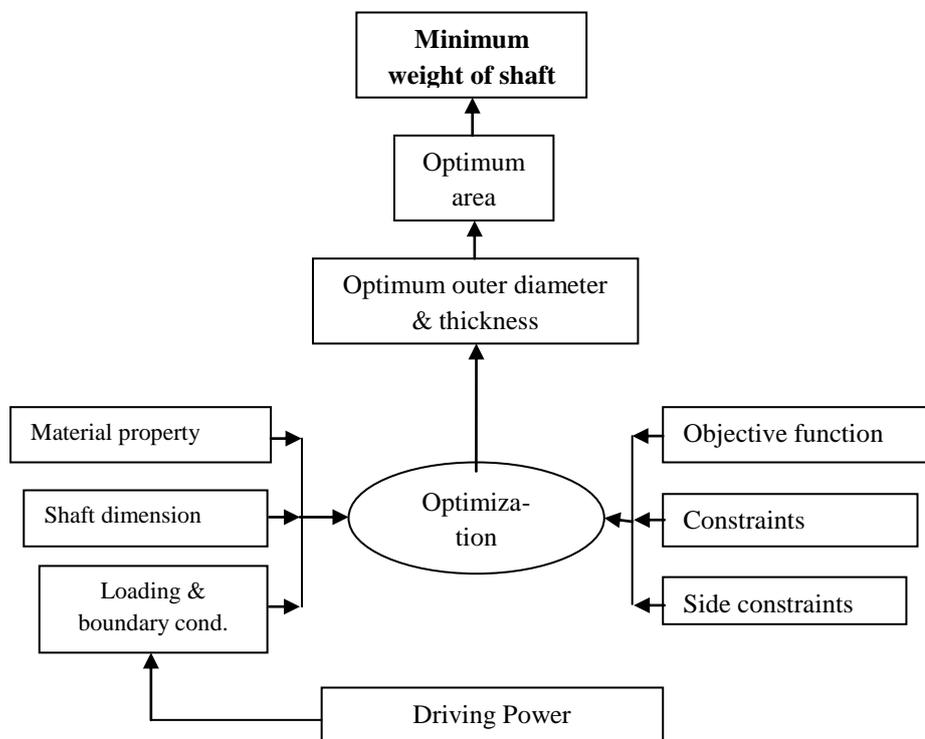


Figure 2. The proposed optimization model

This research developed an optimization model as described in Figure 2. Design variables are outer diameter and thickness of shaft.

Furthermore, the objective function for optimization problems can be written as follows,

$$\text{Minimize : } W = \rho g \pi D t L \quad \dots\dots\dots(5)$$

By assuming that the density of material is constant, the objective function becomes simpler as follows,

$$\text{Minimize : } V = \pi D t L \quad \dots\dots\dots(6)$$

where the meaning of these symbols are:

- V = shaft volume
- D = outer diameter of the shaft
- t = shaft thickness
- L = shaft length

The constraints for this optimization are the induced stress and deflection in the shaft, besides that there is a limit to the thickness of the shaft. These constraints are formulated as follows,

Stress constraint :

$$\sigma_{\max} - \sigma_{\text{ijin}} \leq 0 \quad \dots\dots\dots(7)$$

Deflection constraint:

$$\delta_{\max} - \delta_{\text{ijin}} \leq 0 \quad \dots\dots\dots(8)$$

Shaft thickness constraint:

$$t > 0 \quad \dots\dots\dots(9)$$

### III. APPLICATION

#### III.1. An analyzed shaft

The optimization model developed in this study was applied to a shaft as shown in Figure 3. The material has allowable stress of 89,5MPa for ASM 1144 and allowable deflection of 2 mm.

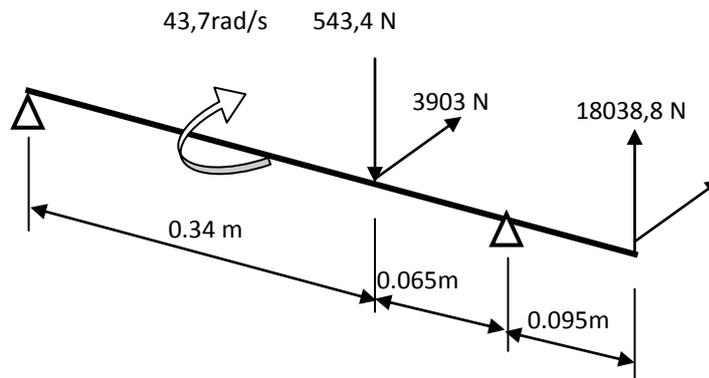


Figure 3. An analyzed shaft

#### III.2. Result and Discussion

The optimization results are shown in table 1 and figure 4 below. It can be seen that the increase in driving power results in an increase in the optimum outer diameter. However the value of driving power and optimum outside diameter are not directly proportional, for the driving power of 3 hp to 5 hp there is an increase in the outer diameter which tends to be parabolic.

The increase in driving power is not directly proportional to the optimum thickness of the shaft. Shaft thickness seems constant, even a decrease in the driving power of 3 hp to 5 hp. Finally, this research has obtained successfully a simple optimization model. The model has been successfully applied to a shaft.

Table 1. Outer diameter and thickness of the shaft resulting from optimization

Power (kW)	Outside Diameter (m)	Thickness (m)	Volume (m <sup>3</sup> )
1	0.0301	0.0045	0.000212
2	0.0346	0.0103	0.000561
3	0.0575	0.0031	0.000275
4	0.0650	0.0027	0.000272
5	0.0650	0.0038	0.000391
6	0.0526	0.0103	0.000849
7	0.0567	0.0106	0.000946
8	0.0572	0.0106	0.000953
9	0.0599	0.0108	0.001013
10	0.0672	0.0080	0.000839

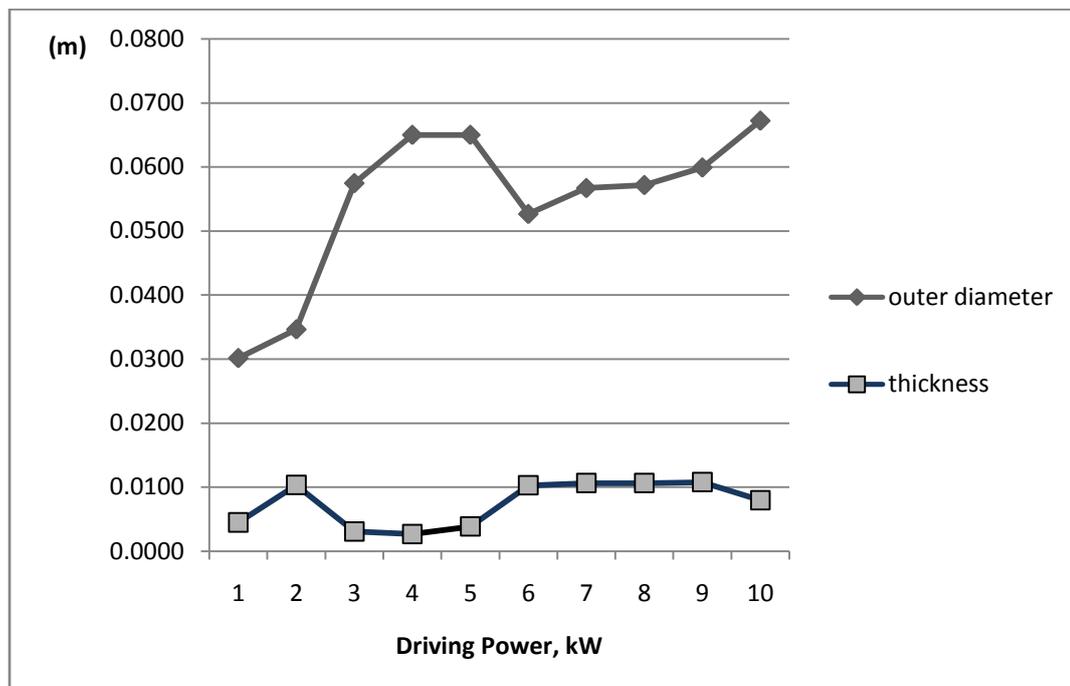


Figure 4. Optimal outer diameter and thickness for variations in driving power

#### IV. CONCLUSION

This study has developed a simple model to optimize the weight of the shaft with various driving power values. The model has been applied successfully to a shaft. The optimum value of the outer diameter and thickness of the shaft have been obtained for driving power of 1 hp to 10 hp. With these values the shaft will have a minimum weight. It was also known that the increase in the value of the driving power against the outer diameter of the shaft was not directly proportional. Practitioners and designers in industry can use the result as a guideline or technical information.

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