Super Convex Dominating Set in the Corona of Graphs

Enrico L. Enriquez
Department of Mathematics
School of Arts and Sciences
University of San Carlos, 6000 Cebu City, Philippines

Abstract: Let $G = (V(G), E(G))$ be a simple graph. A set $S \subseteq V(G)$ is called a convex dominating set of a graph G if for every vertex $u \in V(G) \setminus S$, there exists $v \in S \cap N_G(u)$ and $I_G[S] = S$. It is a super convex dominating set if $N_G(v) \cap (V(G) \setminus S) = \{u\}$. The minimum cardinality of a super convex dominating set of G, denoted by $\gamma_{supc}(G)$, is called the super convex domination number of G. In this paper, we initiate the study of the concept and give some important results. Further, we characterize the super convex dominating set under the corona of two graphs.

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1. Introduction

Domination in graph was introduced by Claude Berge in 1958 and Oystein Ore in 1962 [1]. The paper "Convex sets under some graphs operations," has been defined and studied by Canoy and Garces [2]. Convexity in graphs and convex sets in graphs can be read in [3,4]. The weakly convex and convex domination numbers is found in [5]. The convex doubly connected domination in graphs was introduced by Aunzo and Enriquez [6]. The super dominating sets in graphs was initiated by Lemanska et.al. [7]. Motivated by these parameters, we initiate the study of super convex domination in graphs. For the general concepts, readers may refer to [8].

A graph G is a pair $(V(G), E(G))$, where $V(G)$ is a finite nonempty set called the vertex-set of G and $E(G)$ is a set of unordered pairs $\{u, v\}$ (or simply uv) of distinct elements from $V(G)$ called the edge-set of G. The elements of $V(G)$ are called vertices and the cardinality $|V(G)|$ of $V(G)$ is the order of G. The elements of $E(G)$ are called edges and the cardinality $|E(G)|$ of $E(G)$ is the size of G. If $|V(G)| = 1$, then G is called a trivial graph. If $E(G) = \emptyset$, then G is called an empty graph. The open neighborhood of a vertex $v \in V(G)$ is the set $N_G(v) = \{u \in V(G): uv \in E(G)\}$. The elements of $N_G(v)$ are called neighbors of v. The closed neighborhood of $v \in V(G)$ is the set $N_G[v] = N_G(v) \cup \{v\}$. If $X \subseteq V(G)$, the open neighborhood of X in G is the set $N_G(X) = \cup_{v \in X} N_G(v)$. The closed neighborhood of X in G is the set $N_G[X] = \cup_{v \in X} N_G[v] = N_G(X) \cup X$. When no confusion arises, $N_G[x]$ [resp. $N_G(x)$] will be denoted by $N[x]$ [resp. $N(x)$].

Let G be a simple connected graph. A subset S of a vertex set $V(G)$ is a dominating set of G if for every vertex $v \in V(G) \setminus S$, there exists a vertex $x \in S$ such that xv is an edge of G. The domination number $\gamma(G)$ of G is the smallest cardinality of a dominating set S of G. For more background on dominating sets, the reader may refer to [9-18]. For any two vertices u and v in a connected graph, the distance $d_G(u, v)$ between u and v is the length of a shortest path in G. A u-v path of length $d_G(u, v)$ is also referred to as u-v geodesic. The closed interval $I_G[u, v]$ consist of all those vertices lying on a u-v geodesic in G. For a subset S of vertices of G, the union of all sets $I_G[u, v]$ for $u, v \in S$ is denoted by $I_G[S]$. Hence $x \in I_G[S]$ if and only if x lies on some u-v geodesic, where $u, v \in S$. A set S is convex if $I_G[S] = S$. Certainly, if G is connected graph, then $V(G)$ is convex. A dominating set S which is also convex is called a convex dominating set of G. The convex domination number $\gamma_{con}(G)$ of G is the smallest cardinality of a convex dominating set of G. A convex dominating set of cardinality $\gamma_{con}(G)$ is called a γ_{con}-*set* of G.

A convex dominating set is a super convex dominating set if for every vertex $u \in V(G) \setminus S$, there exists $v \in S \cap N_G(u)$ such that $N_G(v) \cap (V(G) \setminus S) = \{u\}$. The minimum cardinality of a super convex dominating set of G, denoted by $\gamma_{supc}(G)$, is called the super convex domination number of G. In this paper, we initiate the study of super convex dominating set and give some important results. Unless otherwise stated, all subsets of the vertex sets in this paper are assumed to be nonempty.

2. Results

Consider a cycle graph C_6 where $V(C_6) = \{v_1, \ldots, v_6\}$ and $E(C_6) = \{v_1v_2, \ldots, v_5v_6, v_6v_1\}$. If $S = \{v_1, v_2, v_3\}$, then S is convex but not a dominating set since $xv_5 \notin E(C_6)$ for any $x \in S$. If $S = \{v_1, v_2, v_3, v_4\}$, then S is a dominating set but not convex since $I_{C_6}[S] = C_6 \neq S$. Therefore, any subset S of $V(C_6)$ is not convex dominating set of C_6. Hence, we cannot find a super convex dominating set for some connected graph G.

Since $\gamma_{supc}(G)$ does not always exist in a connected nontrivial graph G, we denote by $\mathcal{S}(G)$, a family of all graphs with super convex dominating set. Thus, for the purpose of this study, it is assumed that all connected nontrivial graphs considered in this paper belong to the family $\mathcal{S}(G)$. From the definitions, the following result is immediate.

Remark 2.1 Let G be a connected graph of order $n \geq 2$. Then

(i) $1 \leq \gamma_{supc}(G) \leq n-1$, and
(ii) $\gamma(G) \leq \gamma_{con}(G) \leq \gamma_{supc}(G)$.

Remark 2.2 The super convex dominating set is a super dominating set and a convex dominating set.

It is worth mentioning that the upper bound in Remark 2.1(i) is sharp. For example, $\gamma_{supc}(K_n) = n-1$. The lower bound is also attainable as the following result shows.

Remark 2.3 The $\gamma_{supc}(G) = 1$ if and only if $G = K_2$.

The next result says that the value of the parameter $\gamma_{sups}(G)$ ranges over all positive integers.

Theorem 2.4 Given positive integers k and n such that $n \geq 2$ and $1 \leq k \leq n-1$, there exists a connected graph G with $|V(G)| = n$ and $\gamma_{supc}(G) = k$.

Proof: Consider the following cases:

*Case*1. Suppose $k = 1$.

Let $G = K_2$. Clearly, $|V(G)| = 2 = n$ and $\gamma_{supc}(G) = 1 = k$.

*Case*2. Suppose $2 \leq k < n-1$.

Let $G = H \circ P_1$ where H is a nontrivial connected graph. Let $V(H) = \{a_1, a_2, \ldots, a_k\}$ and $n = 2k$. Then $V(H)$ is a γ_{sups}-set in G (with $2 \leq |V(H)| = k$ and $k < n-1$). Thus, $\gamma_{supc}(G) = k$ and

$$|V(G)| = |V(H \circ P_1)| = |V(H) \cup (\cup_{x \in V(H)} V(P_1^x))| = |V(H)| + \sum_{x \in V(H)} |V(P_1^x)| = k + |V(H)| = k + k = n.$$

*Case*3. Suppose $k = n-1$.

Let $G = K_n$. Then $\gamma_{supc}(G) = n-1 = k$ and $|V(G)| = n$. This proves the assertion. ■

Theorem 2.5 Given positive integers k, m and $n \geq 2$ such that $1 \leq k \leq m \leq n-1$, there exists a connected graph G with $|V(G)| = n$, $\gamma_{supc}(G) = m$, and $\gamma_{con}(G) = k$.

Proof: Consider the following cases:

*Case*1. Suppose $m = n-1$.

*Subcase*1. If $k = m$, then $G = K_2$. Thus, $\gamma_{con}(G) = 1 = k$, $\gamma_{supc}(G) = 1 = 2-1 = n-1 = m$ and $|V(G)| = 2 = n$.

*Subcase*2. If $k < m$, then consider the graph $= K_n$ $(n \geq 3)$. Let $x \in V(G)$. The set $A = V(G) \setminus \{x\}$ is a γ_{supc}-set and $B = \{x\}$ is a γ_{con}-set in G. Thus, $\gamma_{supc}(G) = |A| = |V(G) \setminus \{x\}| = |V(G)| - 1 = n-1 = m$ and $\gamma_{con}(G) = |B| = 1 = k$. Further, $|V(G)| = n$.

*Case*2. Suppose $m < n - 1$.

*Subcase*1. If $k = m$, then let $n = 2m$ and consider the graph $G = H \circ P_1$ where H is a nontrivial connected graph. Let $V(H) = \{a_1, a_2, \dots, a_k\}$. Then the set $A = V(H)$ is a γ_{supc}-set and a γ_{con}-set in G. Thus, $\gamma_{con}(G) = k = m = \gamma_{supc}(G)$. Further,

$$|V(G)| = |V(H \circ P_1)| = |V(H)| + |\cup_{x \in V(H)} V(P_1^x)| = k + k = 2k = 2m = n.$$

*Subcase*2. If $k < m$, then let $m = k + 2(r - 1)$ with $k \geq 2, r \geq 2, n = k + 2r$ and consider the graph $V(G) = V(P_k) \cup V(\overline{K}_a) \cup V(\overline{K}_b)$ with $V(P_k) = \{v_1, \dots, v_k\}$, $V(\overline{K}_a) = \{x_1, \dots, x_r\}$, $V(\overline{K}_b) = \{y_1, \dots, y_r\}$, and
$v_1x_1, \dots, v_1x_r, v_ky_1, \dots, v_ky_r \in E(G)$ as shown in Figure 1.

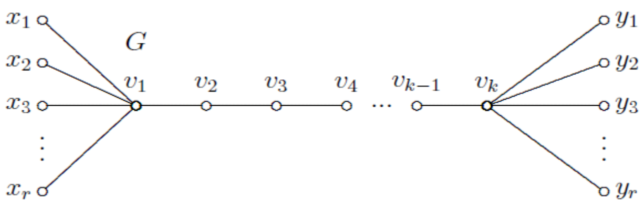

Figure 1: A graph G with $\gamma_{supc}(G) = m$ and $\gamma_{con}(G) = k$

The set $A = V(P_k)$ is a γ_{con}-set in G and $B = A \cup \{x_1, x_2, \dots, x_{r-1}\} \cup \{y_1, y_2, \dots, y_{r-1}\}$ is a γ_{supc}-set in G. Thus, $\gamma_{con}(G) = |A| = k$ and $\gamma_{supc}(G) = k + (r - 1) + (r - 1) = k + 2(r - 1) = m$. Further, $|V(G)| = k + 2r = n$. This proves the assertion. ■

The following result is an immediate consequence of Theorem 2.4.

Corollary 2.6 The difference $\gamma_{supc} - \gamma_{con}$ can be made arbitrarily large.

Proof: Let n be a positive integer. By Theorem 2.5, there exists a connected graph G such that $\gamma_{supc}(G) = n + k$ and $\gamma_{con}(G) = k$. Thus, $\gamma_{sups}(G) - \gamma_{con}(G) = n$, showing that $\gamma_{sups} - \gamma_{con}$ can be made arbitrarily large. ■

Let G and H be graphs of order m and n, respectively. The corona of two graphs G and H is the graph $G \circ H$ obtained by taking one copy of G and m copies of H, and then joining the ith vertex of G to every vertex of the ith copy of H. The join of vertex v of G and a copy H^v of H in the corona of G and H is denoted by $v + H^v$.

We need the following results for the characterization of the corona of two graphs.

Lemma 2.7 Let G and H be nontrivial connected graphs. If $S_v \subset V(H^v)$ is a super convex dominating set of $H^v + v$ for all $v \in V(G)$, then $S = \cup_{v \in V(G)}[V(v + H^v) \setminus (V(H^v) \setminus S_v)]$ is a super convex dominating set in $G \circ H$.

Proof: Suppose S_v is a super dominating set of $v + H^v$ for all $v \in V(G)$. By definition, for each $u \in V(v + H^v)$ there exists $x \in S_v$ such that $N_{v+H^v}(x) \cap [V(v + H^v) \setminus S_v] = \{u\}$. This means that for each $u \in V(v + H^v)$ there exists $x \in \{u\} \cup S_v$ such that $N_{v+H^v}(x) \cap [V(v + H^v) \setminus (\{v\} \cup S_v)] = \{u\}$. Thus, $\{v\} \cup S_v$ is also a super dominating set of $v + H^v$. If S_v is convex, then $\{v\} \cup S_v$ is clearly convex. Thus, $\{v\} \cup S_v$ is a super convex dominating set of $v + H^v$. This clearly implies that, $\cup_{v \in V(G)}(\{v\} \cup S_v)$ is a super convex dominating set of $G \circ H$. Now,

$$\begin{aligned} S &= \bigcup_{v\in V(G)}[V(v+H^v) \setminus (V(H^v) \setminus S_v)] \\ &= \bigcup_{v\in V(G)}[V(v+H^v) \setminus (V(H^v) \cap S_v^c)] \\ &= \bigcup_{v\in V(G)}[V(v+H^v) \cap (V(H^v) \cap S_v^c)^c] \\ &= \bigcup_{v\in V(G)}[V(v+H^v) \cap (V(H^v)^c \cup S_v)] \\ &= \bigcup_{v\in V(G)}[(V(v+H^v) \cap V(H^v)^c) \cup (V(v+H^v) \cap S_v). \end{aligned}$$

Since $V(v+H^v) \cap V(H^v)^c = \{v\}$ and $V(v+H^v) \cap S_v = S_v$, it follows that

$$\begin{aligned} S &= \bigcup_{v\in V(G)}[(V(v+H^v) \cap V(H^v)^c) \cup (V(v+H^v) \cap S_v)] \\ &= \bigcup_{v\in V(G)}[\{v\} \cup S_v]. \end{aligned}$$

Therefore, $S = \bigcup_{v\in V(G)}[V(v+H^v) \setminus (V(H^v) \setminus S_v)] = \bigcup_{v\in V(G)}[\{v\} \cup S_v]$ is a super convex dominating set in $G \circ H$. ■

Lemma 2.8 Let G be a nontrivial connected graph and H be a complete graph. Then a proper subset S of $V(G \circ H)$ is a super convex dominating set in $G \circ H$ if $S = V(G) \cup [\bigcup_{v\in V(G)}(V(H^v) \setminus \{x\})]$ for any $x \in V(H^v)$.

Proof: Suppose that $S = V(G) \cup [\bigcup_{v\in V(G)}(V(H^v) \setminus \{x\})]$ for any $x \in V(H^v)$. Then

$$\begin{aligned} S &= V(G) \cup [\bigcup_{v\in V(G)}(V(H^v) \setminus \{x\})] \\ &= \bigcup_{v\in V(G)}[V(v+H^v) \setminus \{x\}] \\ &= \bigcup_{v\in V(G)}[(\{v\} \setminus \{x\}) \cup (V(H^v) \setminus \{x\})] \\ &= \bigcup_{v\in V(G)}[\{v\} \cup (V(H^v) \setminus \{x\})]. \end{aligned}$$

Since H is complete, $V(H^v) \setminus \{x\}$ is convex and $N_{v+H^v}(y) \cup \{x\} = \{x\}$ for some $y \in V(H^v)$ for all $v \in V(G)$. Thus, $V(H^v) \setminus \{x\}$ is a super convex dominating set of $v + H^v$. Let $S_v = V(H^v) \setminus \{x\}$. Then,

$$\begin{aligned} S &= \bigcup_{v\in V(G)}[\{v\} \cup (V(H^v) \setminus \{x\})] \\ &= \bigcup_{v\in V(G)}(\{v\} \cup S_v). \end{aligned}$$

By the proof of Lemma 2.7, a proper subset S of $V(G \circ H)$ is a super convex dominating set in $G \circ H$. ■

Theorem 2.9 Let G and H be nontrivial connected graphs. Then a subset $S = V(G) \cup \left(\bigcup_{v\in S_G} S_v\right) \cup \left(\bigcup_{x\in V(G)\setminus S_G} S_x\right)$ of $V(G \circ H)$ where $S_G \subseteq V(G)$, $S_v \subset V(H^v)$, and $S_x \subset V(H^x)$ for all $v, x \in V(G)$, is a super convex dominating set in $G \circ H$ if and only if for each $v \in V(G)$, $\{v\} \cup S_v$ is a super convex dominating set of $v + H^v$.

Proof: Let $S_G \subseteq V(G)$, $S_v \subset V(H^v)$, and $S_x \subset V(H^x)$ for all $v, x \in V(G)$. Suppose that a subset $S = V(G) \cup \left(\bigcup_{v\in S_G} S_v\right) \cup \left(\bigcup_{x\in V(G)\setminus S_G} S_x\right)$ of $V(G \circ H)$ is a super convex dominating set dominating set of $G \circ H$. Consider the following cases:

*Case*1. Suppose $S_G = V(G)$. Then

$$\begin{aligned} S &= V(G) \cup \left(\bigcup_{v\in V(G)} S_v\right) \cup \left(\bigcup_{x\in\emptyset} S_x\right) = V(G) \cup \left(\bigcup_{v\in V(G)} S_v\right) \\ &= \left(\bigcup_{v\in V(G)}\{v\}\right) \cup \left(\bigcup_{v\in V(G)} S_v\right) = \bigcup_{v\in V(G)}(\{v\} \cup S_v). \end{aligned}$$

Suppose $\{v\} \cup S_v$ is not a convex set in $v + H^v$. Then S is not convex contrary to our assumption that S is convex. Thus, $\{v\} \cup S_v$ must be convex in $v + H^v$ for all $v \in V(G)$. Similarly, since S is a super dominating set, $\{v\} \cup S_v$ must be a super dominating set in $v + H^v$.

*Case*2. Suppose $S_G \neq V(G)$. If $v = x$, then

$$S = V(G) \cup \left(\bigcup_{v\in S_G} S_v\right) \cup \left(\bigcup_{v\in V(G)\setminus S_G} S_v\right)$$

$$= V(G) \cup (\cup_{v\in V(G)} S_v) = \cup_{v\in V(G)}(\{v\} \cup S_v).$$

By Case1, $\{v\} \cup S_v$ is a super dominating set in $v + H^v$. Now, if $v \neq x$, then

$$\begin{aligned} S &= V(G) \cup \left(\cup_{v\in S_G} S_v\right) \cup (\cup_{x\in V(G)\setminus S_G} S_x) \\ &= [S_G \cup (V(G) \setminus S_G)] \cup \left(\cup_{v\in S_G} S_v\right) \cup (\cup_{x\in V(G)\setminus S_G} S_x) \\ &= S_G \cup \left(\cup_{v\in S_G} S_v\right) \cup (V(G) \setminus S_G) \cup (\cup_{x\in V(G)\setminus S_G} S_x) \\ &= \left(\cup_{v\in S_G}\{v\}\right) \cup \left(\cup_{v\in S_G} S_v\right) \cup \left(\cup_{x\in V(G)\setminus S_G}\{x\}\right) \cup (\cup_{x\in V(G)\setminus S_G} S_x) \\ &= \left(\cup_{v\in S_G}(\{v\} \cup S_v)\right) \cup (\cup_{x\in V(G)\setminus S_G}(\{x\} \cup S_x)). \end{aligned}$$

If $\{v\} \cup S_v$ is not a super convex dominating set in $v + H^v$ for some $v \in S_G$, then S is not a super convex dominating set is clear. Similarly, if $\{x\} \cup S_x$ is not a super convex dominating set in $x + H^x$ for some $x \in V(G) \setminus S_G$, then S is not a super convex dominating set. In either case, our assumption that S is super convex dominating set is contradicted. This implies that $\{y\} \cup S_y$ must be a super convex dominating set in $y + H^y$ for all $y \in V(G)$.

For the converse, suppose that for each $v \in V(G)$, $\{v\} \cup S_v$ is a super convex dominating set of $v + H^v$. Then $\cup_{v\in V(G)}(\{v\} \cup S_v)$ is a super convex dominating set of $G \circ H$. Let $S = \cup_{v\in V(G)}(\{v\} \cup S_v)$. Now,

$$\begin{aligned} S &= \cup_{v\in V(G)}(\{v\} \cup S_v) = \left(\cup_{v\in V(G)}\{v\}\right) \cup (\cup_{v\in V(G)} S_v) \\ &= V(G) \cup (\cup_{v\in V(G)} S_v) = V(G) \cup \left(\cup_{v\in S_G} S_v\right) \cup (\cup_{x\in V(G)\setminus S_G} S_x), \end{aligned}$$

where $S_G \subseteq V(G)$, $S_v \subset V(H^v)$, and $S_x \subset V(H^x)$ for all $v, x \in V(G)$. ■

Remark 2.10 If G (or H) is trivial then $S = \{v\} \cup S_v$ (or $V(G)$) is a super convex dominating set of $G \circ H$ for all $v \in V(G)$, where $\{v\} \cup S_v$ is a super convex dominating set of $V(v + H^v)$.

Corollary 2.11 Let G and H be nontrivial connected graphs. Then $\gamma_{supc}(G \circ H) = \gamma_{supc}(v + H^v)|V(G)|$.

Proof: Suppose that $S = V(G) \cup \left(\cup_{v\in S_G} S_v\right) \cup (\cup_{x\in V(G\setminus S_G)} S_x)$ of $V(G \circ H)$ where $S_G \subseteq V(G)$, $S_v \subset V(H^v)$, and $S_x \subset V(H^x)$ for all $v, x \in V(G)$, is a super convex dominating set in $G \circ H$. Then for each $v \in V(G)$, $\{v\} \cup S_v$ is a super convex dominating set of \$$v + H^v$ by Theorem 2.8. Thus, $\gamma_{supc}(G \circ H) \leq |S|$. Now,

$$\begin{aligned} \gamma_{supc}(G \circ H) &\leq |V(G) \cup \left(\cup_{v\in S_v} S_v\right) \cup (\cup_{x\in V(G)\setminus S_G} S_x)| \\ &= |V(G)| + (\textstyle\sum_{v\in S_G}|S_v|) + (\sum_{x\in V(G)\setminus S_G}|S_x|) \\ &= |V(G)| + (\textstyle\sum_{v\in\setminus S_G}|S_v|) + (\sum_{x\in V(G)\setminus S_G}|S_v|) \\ &\qquad \text{where } |S_v| = |S_x| \text{ for all } S_v \subset V(H^v), S_x \subset V(H^x) \\ &= |V(G)| + (\textstyle\sum_{v\in V(G)}|S\_v|) \text{ for all } S_v \subset V(H^v) \\ &= |V(G)| + |V(G)||S_v| \text{ for all } S_v \subset V(H^v) \\ &= |(1 + |S_v|)|V(G)| \text{ for all } S_v \subset V(H^v). \end{aligned}$$

Since $\{v\} \cup S_v$ is a super convex dominating set of $v + H^v$ for all $v \in V(G)$, it follows that $\gamma_{supc}(G \circ H) \leq \gamma_{supc}(v + H^v)|V(G)|$. Now, suppose that S^o is a minimum super convex dominating set of $G \circ H$.
Then

$$\begin{aligned} \gamma_{supc}(G \circ H) = |S^o| &= |V(G) \cup \left(\cup_{v\in S_G} S_v^o\right) \cup (\cup_{x\in V(G)\setminus S_G} S_x^o)| \\ &= |V(G)| + (\textstyle\sum_{v\in S_G}|S_v^o|) + (\sum_{x\in V(G)\setminus S_G}|S_x^o|) \\ &= |V(G)| + (\textstyle\sum_{v\in S_G}|S_v^o|) + (\sum_{x\in V(G)\setminus S_G}|S_v^0|) \\ &\qquad \text{where } |S_v^o| = |S_x^o| \text{ for all } S_v^o \subset V(H^v), S_x^o \subset V(H^x) \\ &= |V(G)| + (\textstyle\sum_{v\in V(G)}|S_v^o|) \text{ for all } S_v^o \subset V(H^v) \\ &= |V(G)| + |V(G)||S_v^o| \text{ for all } S_v^o \subset V(H^v) \\ &= |(1 + |S_v^o|)|V(G)| \text{ for all } S_v^o \subset V(H^v). \end{aligned}$$

Since $\{v\} \cup S_v^o$ is a super convex dominating set of $v + H^v$ for all $v \in V(G)$, it follows that $\gamma_{supc}(G \circ H) = |(1 + |S_v^o|)|V(G)| \geq \gamma_{supc}(v + H^v)|V(G)|$. ■

3. Conclusion and Recommendations

In this work, we introduced a new parameter of domination in graphs, the super convex domination in graphs. The super convex domination in corona of two graphs was characterized. The exact super convex domination number resulting from this binary operation of two graphs was computed. This study will pave a way to new research such bounds and other binary operations of two graphs. Other parameters involving super convex domination in graphs may also be explored. Finally, the characterization of a super convex domination in graphs and its bounds is a promising extension of this study.

References

[1] O. Ore. *Theory of Graphs*. American Mathematical Society, Provedence, R.I., 1962.

[2] S.R. Canoy Jr., I.J.L. Garces, *Convex Sets Under Some Graphs Operations*, Graphs Comb. 18(2002), 787-793.

[3] F. Harary and J. Nieminen, *Convexity in graphs*. J. Differ Geom. 16(1981), 185-190.

[4] G. Chartrand, and P. Zhang, *Convex sets in graphs*, Congressus Numerantium, 136 (1999), 19-32.

[5] M. Lemanska, *Weakly convex and convex domination numbers*, Opuscula Mathematica, 24 (2004), 181-188.

[6] R.T. Aunzo, and E.L. Enriquez, *Convex Doubly Connected Domination in Graphs*, Applied Mathematical Sciences, Vol. 9, 2015, no. 135, 6723-6734.

[7] M. Lemanska, V. Swaminathan, Y. B. Venkatakrishnan, R. Zuazua, *Super Dominating sets in graphs*, Proceedings of the National Academy of Sciences, Vol. 85, 2015, no. 3, 353-357.

[8] G. Chartrand and P. Zhang, *A First Course in Graph Theory*, Dover Publication, Inc., New York, 2012.

[9] E.J. Cockayne, and S.T. Hedetniemi, Towards a theory of domination in graphs, *Networks*, (1977) 247-261.

[10] C. M. Loquias, and E. L. Enriquez, *On Secure and Restrained Convex Domination in Graphs*, International Journal of Applied Engineering Research, Vol. 11, no. 7 (2016), 4707-4010.

[11] C. M. Loquias, E. L.Enriquez, and J. Dayap. *Inverse Clique Domination in Graphs.* Recoletos Multidisciplinary Research Journal. Vol. 4, No. 2 (2017), pp 23-34.

[12] E.L.Enriquez, and S.R. Canoy, Jr., *Secure Convex Domination in a Graph*. International Journal of Mathematical Analysis, Vol. 9 (2015), no. 7, 317-325.

[13] E.L. Enriquez, and S.R. Canoy,Jr., *On a Variant of Convex Domination in a Graph*. International Journal of Mathematical Analysis, Vol. 9, 2015, no. 32, 1585-1592.

[14] E. L. Enriquez, and S. R. Canoy,Jr., *Restrained Convex Dominating Sets in the Corona and the Products of Graphs*. Applied Mathematical Sciences, Vol. 9, 2015, no. 78, 3867 - 3873.

[15] E.M. Kiunisala, and E.L. Enriquez, *Clique Secure Domination in Graphs.* Global Journal of Pure and Applied Mathematics, Vol. 12, No. 3 (2016), pp. 2075–2084.

[16] M. P. Baldado, Jr. and E. L. Enriquez, *Super Secure Domination in Graphs*, International Journal of Mathematical Archive-8(12), 2017, pp. 145-149.

[17] E. L. Enriquez, *Super Restrained Domination in the Corona of Graphs*, International Journal of Latest Engineering Research and Applications, 3(5), 2018, pp. 1 – 6.

[18] E. L. Enriquez, *Super Fair Dominating Set in Graphs*, Journal of Global Research in Mathematical Archives, 6(2), 2019, pp 8 – 14.