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Motivating Students Learning with Dynamic Modeling

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Abstract: Dr. Winkel, director of SIMIODE promotes using computer modeling to teach Differential Equations. Dr. Panoff, director of SHORDOR.COM promotes using computer modeling to unzipped student's potential. I had a chance to attend both SIMIODE and SHORDOR workshops. I learned how to develop models that can be used in classroom and how to use models to teach classes. I used Dr. Winkel M and M model at all my classes. Different level students have different reports. I started using modeling to motivate student at all level classes. I used Dr. Panoff Have = Had + change model to inspire student working on undergraduate research in the topic of dynamic modeling of real-world cases. Since all students have different background. Dynamic modeling is the best way to encourage students working on interdisciplinary field. Differential Equations are the best way to describe the change of any phenomena. However differential equation is a spooky term for many students. If we use difference quotient that students learned in College Algebra to approximate derivative, then even lower-level students can enjoy differential equation models.

This paper is based on my presentation at Joint Mathematics Meeting at Baltimore, MD in January 2019. At this paper, I will introduce some models that I used in my classes.

Keywords: Dynamic Modeling, SIMIODE, SHORDOR, M and M model, Cinderella Model

Introduction

Many students complain that those mathematics skills they learned in classroom have nothing to do with their career after graduation. Many mathematicians tried hard to develop projects that can link classroom mathematics to real life issues. Many real-world situations are related to the change of rate of a quantity that is proportional to the amount of current amount or other quantity. Such might be the case for a population of people, rabbits, or bacteria. When money is compounded continuously, the rate of change of the amount is also proportional to the amount present. For a radioactive element, the number of radioactive decays at a rate proportional to the amount present. All those situations can be modeled by differential equations. However, differential equations are very scary terms to many students. "Difference quotient" is more user-friendly term to students since it is the concept students learned in high school mathematics, and College Algebra classes. There are many software for dynamic modeling. Excel is one that every student already knows basic Excel skills. At this paper I will use discrete different quotient to approximate continuous differential equations and use Excel instead of other modeling software.

Model and Analysis

1. Steps of the Modeling

First of all, I will let student know that there is no correct model. But there is always a better model. Depending on personal background, one can always create a mathematical model initially. Later one can modify its initial model and get a better realistic model. Therefore, the modeling process is cyclic and closely parallels in scientific method. The modeling process is as following:

a) Analyze the problem: At this step, students need to identify major variables and parameters and the relationship between them. Also students need to identify what assumptions do they need for their model. It is better to draw a diagram to show the model. Students can download shareware Vensim for education purpose. Vensim is a modeling software. I only expect student use its drawing tool.

b) Formulate the model: From the diagram, students can find out the equation between variables and parameters. Write approximation difference equations.

c) Enter those equations into Excel worksheet to get graphic solutions.

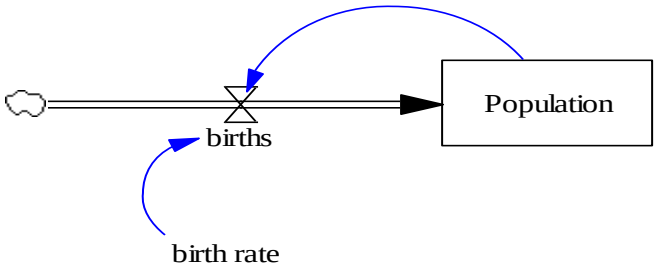
d) Verify and interpret the model solutionif it is realistic.

e) Add more conditions and go through step a), b), c) and d) to revise the model!

2. Population Models

As an example of creating models, I shall start the simple population model.

This model is a study of population growth based on a growth factor over time. The easiest way to visualize a population growth model is to assume that some proportion of the population reproduces during each time step and some proportion dies. The difference between the two is the growth factor. This model simulates a simplified model where the population increases is proportional to the current population with proportional growth constant b.

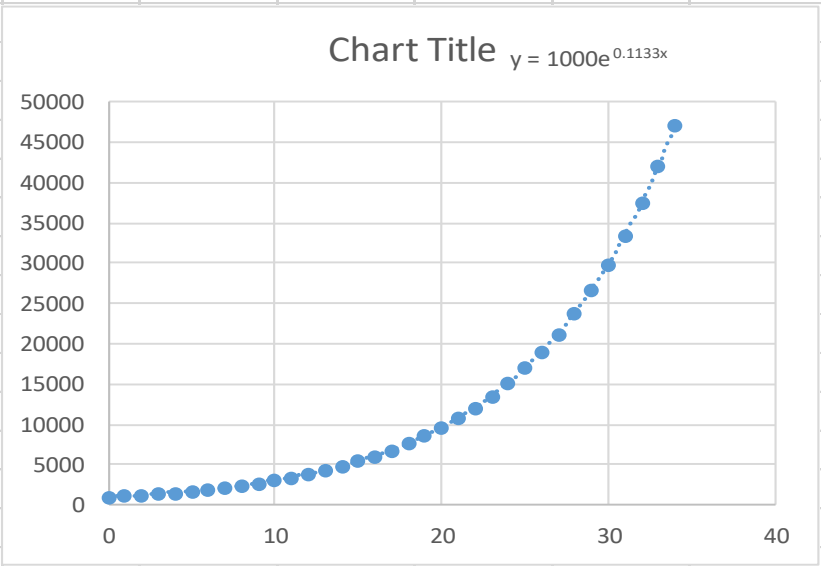


From the diagram, we know the change of the population is birth rate * current population.

The mathematics formula is $\frac{dP}{dt} = b * P$, where $\frac{dP}{dt}$ is the change of population and b is the growth constant. We can use average rate of change to approximate instantaneous rate of change. Therefore we have difference equation $\frac{P(t+\Delta t)-p(t)}{\Delta t} = b * P(t)$. The equation that we enter to Excel is $P(t + \Delta t) = P(t) + \Delta t * b * P(t)$.

Assume initial population is 1000, growth rate is 0.6, and Δt = 0.2, we have the graphic solution:

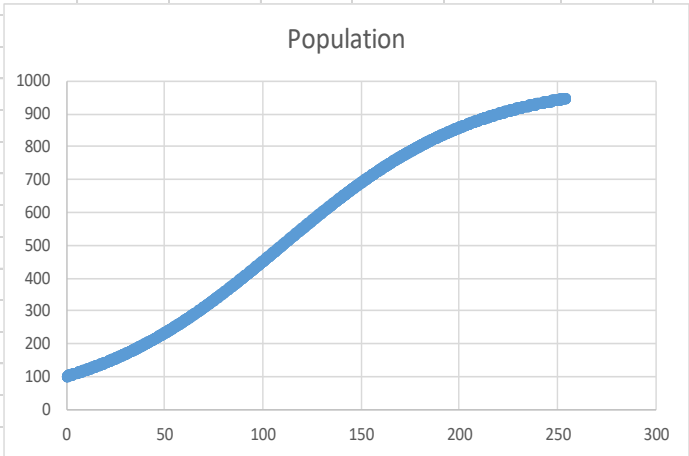
Simple population Growth Model: P(t + delta_t) = p(t) + delta_t*b*p(t)							
Iteration	Population		b =	0.6	delta-t	0.2	
0	1000			60		20	
1	1120						
2	1254						
3	1405						
4	1574						
5	1762						
6	1974						
7	2211						
8	2476						
9	2773						
10	3106						
11	3479						
12	3896						
13	4363						
14	4887						
15	5474						
16	6130						
17	6866						



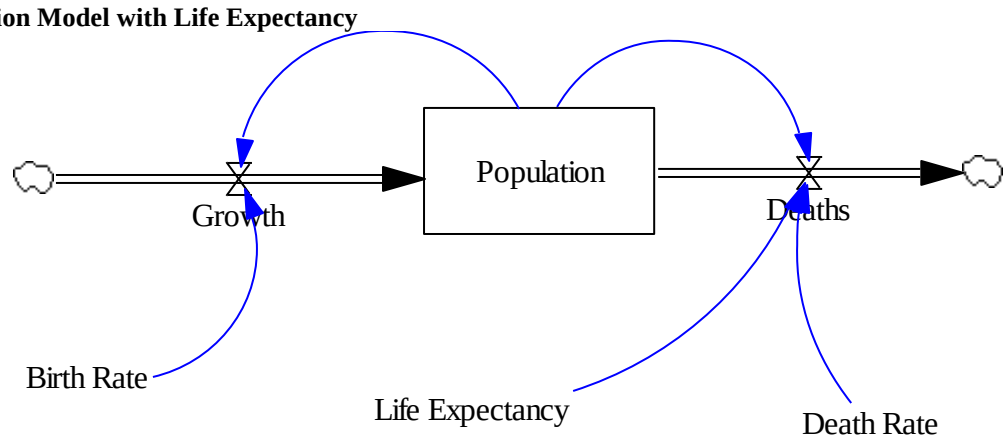
Certainly this model is not realistic since the population cannot grow infinitely. We can revise our model by adding competition for limit resources. If the resources of a community can only support maximal population M, the model becomes Logistics Growth Model. Initially population growth rate is the birth rate.

When population is approaching the limit M, the growth rate become near to zero. We can use the equation birth rate b*(1 - $\frac{P}{M}$) to represent population growth rate and the equation of the change of population is $\frac{dp(t)}{dt} = b * (1 - \frac{P(t)}{M}) * p(t)$. The approximation formula becomes $\frac{P(t+\Delta t)-p(t)}{\Delta t} = b * (1 - \frac{P(t)}{M}) * P(t)$, With initial population 100, birth rate 0.02 and maximal capacity M = 1000, The following Excel solution tells it is similar to Logistic Growth model.

Time	Population	birth	death		Birth Rate	0.02	Death Rate		Maximal P	1000
0	100									
0.1	100.18									
0.2	100.360288									
0.3	100.540864									
0.4	100.721729									
0.5	100.902883									
0.6	101.084326									
0.7	101.266058									
0.8	101.448081									
0.9	101.630393									
1	101.812997									
1.1	101.995891									
1.2	102.179076									
1.3	102.362553									
1.4	102.546322									
1.5	102.730384									
1.6	102.914737									
1.7	103.099384									
1.8	103.284324									



Population Model with Life Expectancy

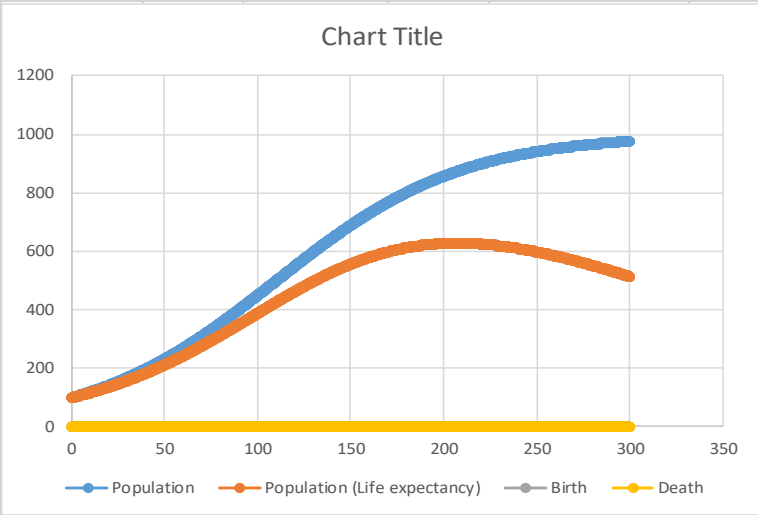


Life expectancy will change population growth. If life expectancy is short, then population growth become small. If life expectance is long, then population growth is high. Life expectancy may affect growth rate.

Growth Rate = Birth rate – $\frac{1}{1+x^2}$*Birth rate, where x is the life expectance. When x = 0, there is no population growth. When x is very large, the population growth rate is very close to birth rate.

The Excel Worksheet shows the population growth with maximal capacity 1000, and life expectancy 20 assuming initial population 100, birth rate 0.02

Time	Population	Population (Life e:		
0	100	100		
0.1	100.2	100.1750623		
0.2	100.38	100.3300748		
0.3	100.5603199	100.4853623		
0.4	100.7409276	100.6408926		
0.5	100.9218235	100.7966661		
0.6	101.1030079	100.9526829		
0.7	101.2844811	101.1089434		
0.8	101.4662435	101.2654478		
0.9	101.6482954	101.4221964		
1	101.8306371	101.5791894		
1.1	102.0132689	101.7364271		
1.2	102.1961912	101.8939097		
1.3	102.3794043	102.0516375		
1.4	102.5629086	102.2096107		
1.5	102.7467043	102.3678296		
1.6	102.9307918	102.5262945		
1.7	103.1151715	102.6850056	0.184672088	0.025715

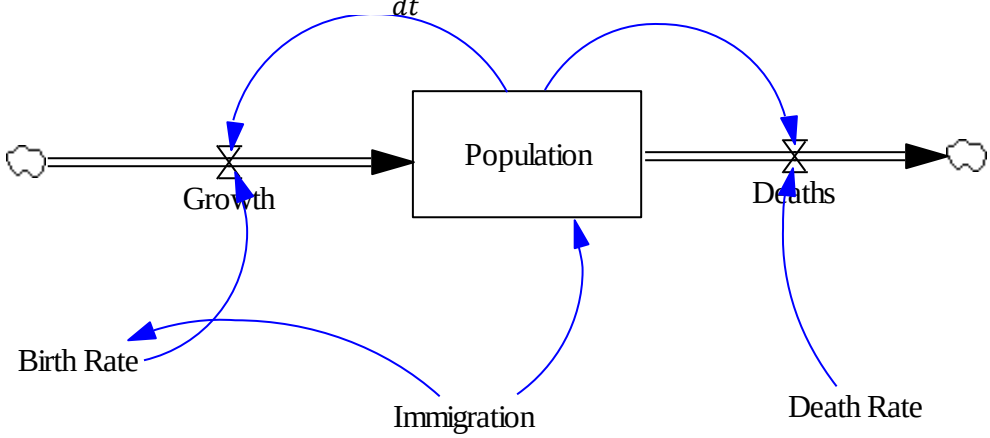


The growth rate here is only our assumption. For real growth rate that related to life expectance, we need to collect real data and find out.

Population Model with Immigration

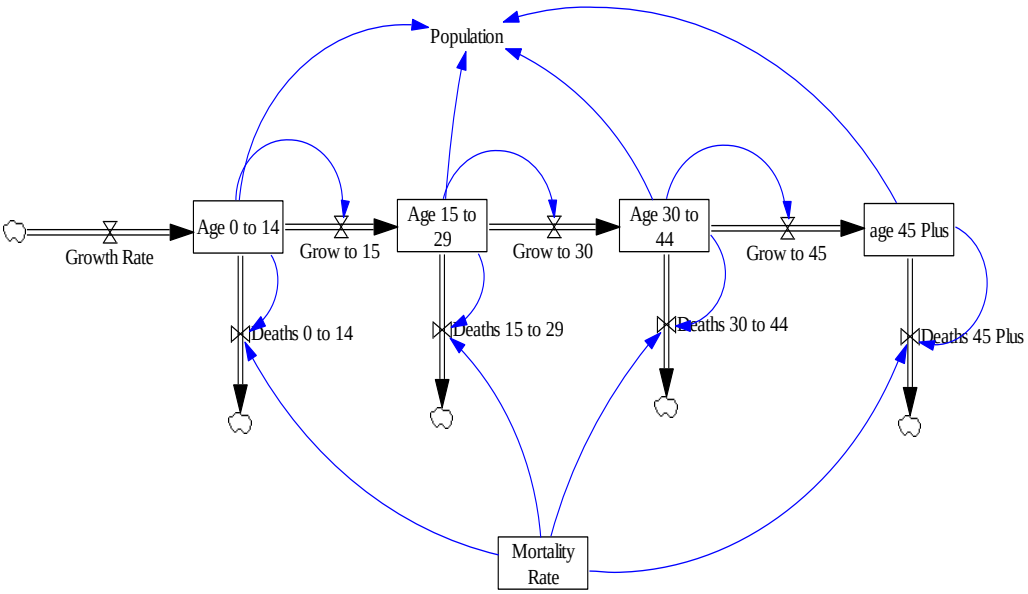
Assume that government allows at most 2% of current population immigrants to the community every period. We assume Birth rate is b, Population is P, and Death rate is d. We don't know exactly how many immigrants will move in the community. We use random *0.02 *P to estimate immigration population per period. The population growth rate, r = b – d, and the change of population is

$$\frac{dP}{dt} = rP + randon * 0.02 * P$$



Population Model with Different aged groups

All previous model is not realistic, becausewe assume all people have the same fertility rate and death rate. To make our model better, we divide population into 4 groups by their ages, P15, P30, P45, and P60, where P15 is the group of people who are younger than 15 years old, P30 is the group of people whose ages are between 15 and 30, P45 is the group of people whose ages are between 30 and 45, and P60 are the people whose ages are older than 45. People in each group has different death rate, fertility rate and life expectancy. Infant mortality is counted death rate in the group P15.



The sequence of population models we introduce above is to let students know that we can always refine our model and get a better one.

3. Dr. Bob Panoff's Universal Formula Have = Had + Change Model

Way back to 20 century I learned this universal formula Have = Had + change from Dr. Bob Panoff, CEO of Shodor.org. This formula can be used at most scenarios in real world. When we want to buy a house (Have), we need to save down Payment (had) and pay monthly payment (change). When we want to learn new skills (Have), we need to satisfy prerequisite (Had) and gain new skills (change).

If we use mathematics sentence to interpret this formula, we have

$$Q(t + h) = Q(t) + \Delta Q$$

But the Change is proportion to current background and the time unit. We can write the equation as

$$Q(t + h) = Q(t) + k*Q(t)*h \text{ ..... (a)}$$

and $\frac{Q(t+h)-Q(t)}{h} = k*Q(t)$ ..... (b)

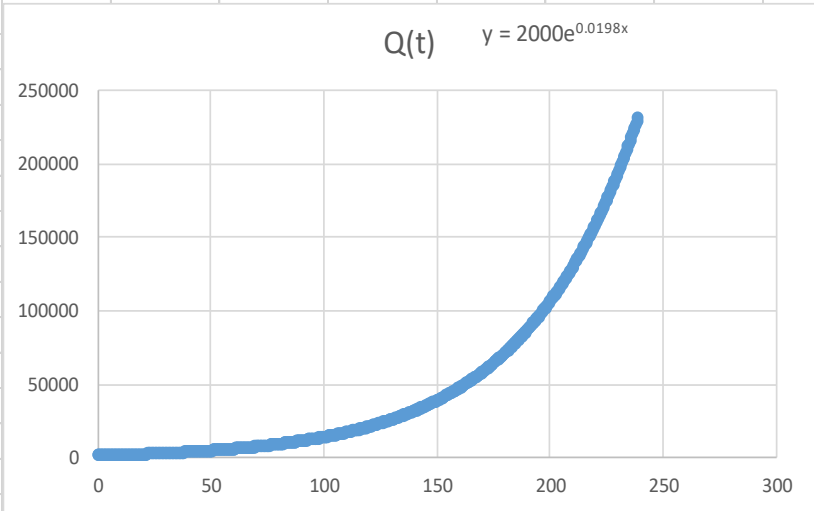
When time period h approaches 0, we have an exponential model differential equation.

$$\frac{dQ(t)}{dt} = k*\ Q(t) \text{ ..... (c)}$$

and we can have a solution $Q(t) = Q(0)*e^{kt}$ ..... (d)

This is the example of typical exponential growth and decay model or balance of Compounding Interest when Compound continually at College Algebra or Precalculus classes. Calculus students can solve equation (C) using separate variables method to get algebraic solution (d). Lower level students can use Scatter Plot to explore solution (d)

time	Q(t)	h =	0.5	k =	0.02
0	2000				
0.5	2020				
1	2040.2				
1.5	2060.602				
2	2081.208				
2.5	2102.02				
3	2123.04				
3.5	2144.271				
4	2165.713				
4.5	2187.371				
5	2209.244				
5.5	2231.337				
6	2253.65				
6.5	2276.187				
7	2298.948				
7.5	2321.938				
8	2345.157				

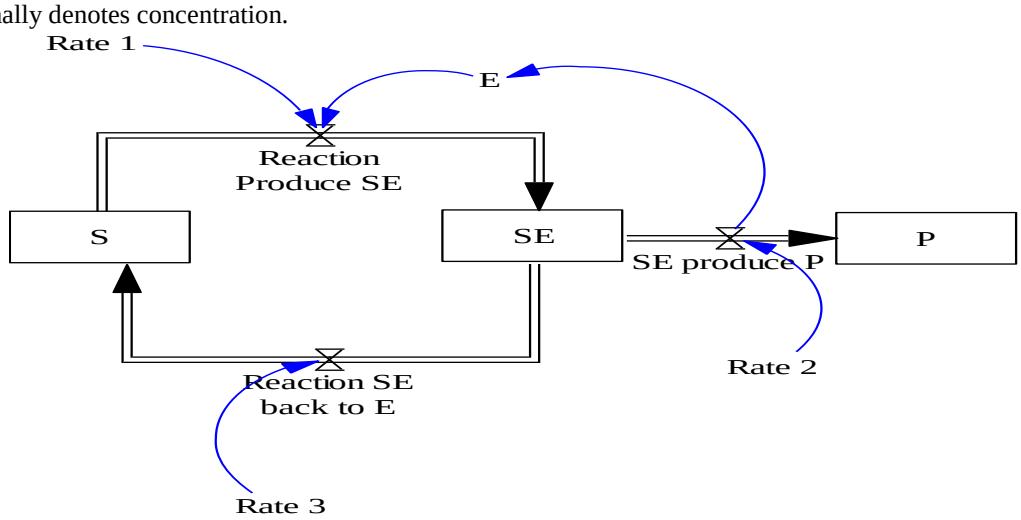


4. Enzyme Kinetics: Basic Enzyme Reaction

One of the most basic enzymatic reactions, first found by Michaelis and Menten (1913), involves a substrates S reacting with an enzyme E to form a complex SE which in turn is converted into a product P and the enzyme. We can represent this schematically by

$$S + E \underset{k3}{\overset{k1}{\longleftrightarrow}} SE \xrightarrow{k2} P + E \quad \ldots\ldots\ldots\ldots (1)$$

Here k1, k2 and k3 are constant parameters associated with the rates of reaction. The Law of Mass Action says that the rate of a reaction is proportional to the product of the concentrations of the reactants. Let us denote the concentrations of the reactants in (1) by lower case letters s =[S], e = [E], c = [SE], p = [P], where [ ] traditionally denotes concentration.



The change of s, e, c and p are as following nonlinear differential equations

$$\frac{ds}{dt} = -k_1 es + k_3 c, \ (2)$$

$$\frac{de}{dt} = -k_1 es + (k_2 + k_3)c, (3)$$

$$\frac{dc}{dt} = k_1 es - (k_2 + k_3)c, (4) \text{ and}$$

$$\frac{dp}{dt} = k_2 c \ (5)$$

Equation (2) is simply the statement that the rate of change of the concentration [S] is made up of a loss rate proportional to [S][E] and a gain rate proportional to [SE]. Suppose the initial conditions are given by

$$s(0) = s_0,\ e(0) = e_0,\ c(0) = 0 \text{ and } p(0) = 0 \quad (6)$$

The solutions of equations (2), (3), (4), (5) and (6) then give the concentrations and hence the rates of the reactions, as functions of time.

Adding the equation (3) add the equation (4), we have

$$\frac{de}{dt} + \frac{dc}{dt} = 0 \Rightarrow e(t) + c(t) = e_0$$

$$e(t) = e_0 - c(t)$$

Equation (2) becomes $\frac{ds}{dt} = -k_1\big(e_0 - c(t)\big)s(t) + k_3 c = -k_1 e_0 s(t) + (k_1 s + k_3)c(t).$

With this, the system of differential equation reduces to only three, for s, c and p, namely.

$$\frac{ds}{dt} = -k_1 e_0 s + (k_1 s + k_3)c, \ (7)$$

$$\frac{dc}{dt} = k_1 e_0 s - (k_1 s + k_3 + k_2)c, \ (8)$$

$$\frac{dp}{dt} = k_2 c \ (9)$$

With initial conditions $s(0) = s_0$, $c(0) = 0$, and $p(0) = 0$.

It is very complicate to solve these three simultaneous differential equations. With help of Excel we can get numerical and graphical solutions.

The approximation formula we entered to Excel are.

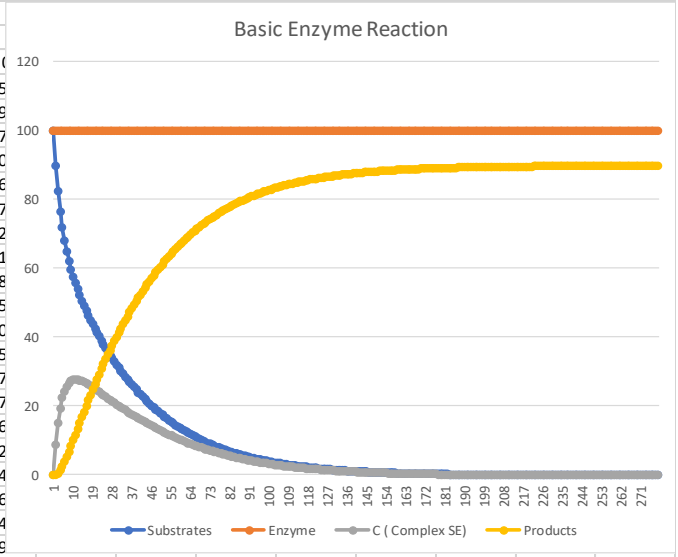
$$s(t + \Delta t) = s(t) + \Delta t(-k_1 e_0 s + (k_1 s + k_3)c)$$

$$c(t + \Delta t) = c(t) + \Delta t(k_1 e_0 s - (k_1 s + k_3 + k_2)c)$$

$$p(t + \Delta t) = p(t) + \Delta t(k_2 c)$$

The Excel solution is as following:

Time	Substrates	Enzyme	C (Complex SE)	Products	k1 =	0.005	K2 =	0.3	K3 =	0.4	Delta t =	0.2
0	100	100	0									
0.2	90	100	9									
0.4	82.53	100	15.183	(								
0.6	76.744693	100	19.47879631	1.45								
0.8	72.1234216	100	22.46921275	2.619								
1	68.329173	100	24.53588376	3.967								
1.2	65.135643	100	25.93790769	5.440								
1.4	62.3865937	100	26.85577768	6.996								
1.6	59.971837	100	27.41771202	8.607								
1.8	57.8123608	100	27.71617786	10.2								
2	55.8507566	100	27.81865095	11.91								
2.2	54.0448657	100	27.77483369	13.58								
2.4	52.363453	100	27.62161511	15.25								
2.6	50.7832001	100	27.38654586	16.90								
2.8	49.2865802	100	27.09031102	18.55								
3	47.8603358	100	26.74851227	20.17								
3.2	46.494376	100	26.37296538	21.7								
3.4	45.1809702	100	25.97265268	23.36								
3.6	43.914155	100	25.55442716	24.92								
3.8	42.6892948	100	25.12353576	26.4								
4	41.5027542	100	24.68401015	27.96								
4.2	40.351654	100	24.23895972	29.44								
4.4	39.2336875	100	23.79079199	30.89								
4.6	38.1469826	100	23.34137888	32.32634								

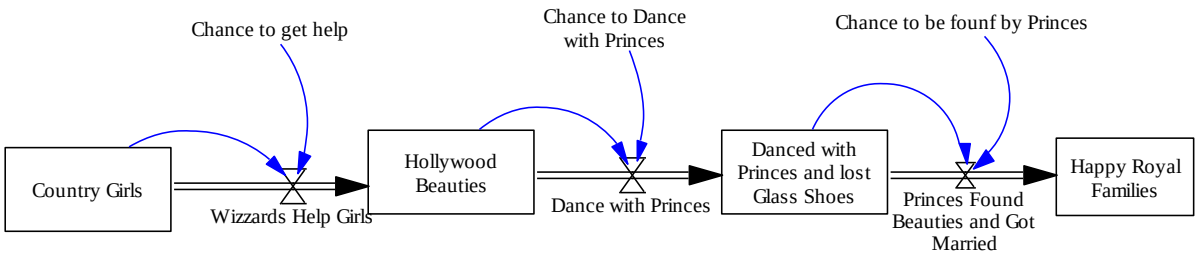


5. Cinderella Model

We can also create mathematics model for children's stories such as Sleeping Beauty, Frog Prince, and Cinderella.

Story of Cinderella has been a most well-known fiction in little kids' world. We can summarize the story as following: A group of country girls who wanted to marry princes. By chance, some girls got wizards' help became Hollywood beauties. By chance some of those beauties danced with princes and lost glass high hills. By chance princes found those beauties through their glass hill hills and married them.

We can draw a diagram to show this story and make a logical mathematical model.



Notations:
G(t): Quantity of Country girls,
H(t): Quantity of Hollywood beauties,
D(t): Quantity of girls who danced with princes,
R(t): Quantity of royal families
r: chance to get help.
s: Chance to dance with princes.
t: Chance to marry princes
The change of G(t) can be described by

$$\frac{dG}{dt} = -rG$$

Since H(t) will increase from the change of G(t) and decrease to D(t), so it can be described by

$$\frac{dH}{dt} = rG - sH$$

The same reason D(t) will gain from H(t) and lose some to R(t). The rate of change will be

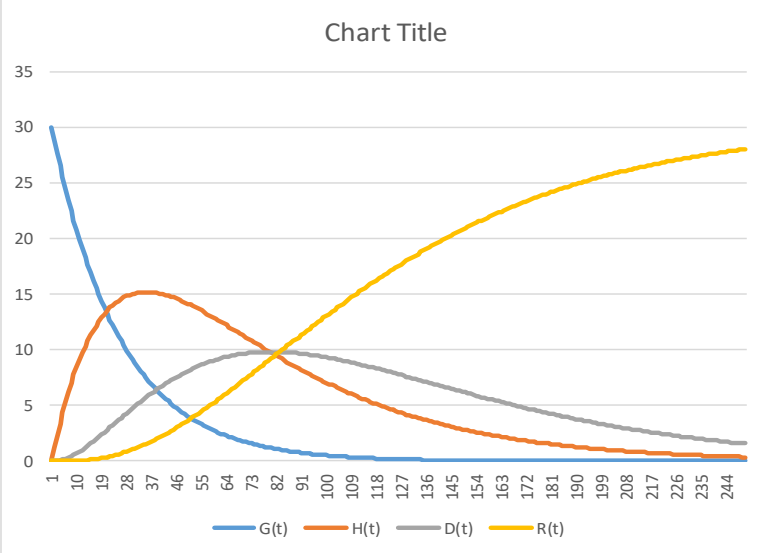
$$\frac{dD}{dt} = sH - tD$$

Finally, the R(t) is a stable quantity who only gain from D(t), so it can be described as

$$\frac{dR}{dt} = tD$$

Initially if we assume G(0) = 30, H(0) = 0, D(0) = 0, R(0) = 0, r = 0.2, s = 0.1, and t = 0.1, using Excel we can get numerical and graphic solutions of these system of differential equations.

G(t)	H(t)	D(t)	R(t)	Parameters		Delta-t	0.2
30	0	0	0	r: Chance to get help	0.2		
28.8	1.2	0	0	s: chance to dance with prir	0.1		
27.648	2.328	0.024	0	t: Chance to Get Married	0.1		
26.54208	3.38736	0.07008	0.00048				
25.4804	4.381296	0.136426	0.001882				
24.46118	5.312886	0.221323	0.00461				
23.48273	6.185075	0.323154	0.009037				
22.54342	7.000683	0.440393	0.0155				
21.64169	7.762407	0.571599	0.024308				
20.77602	8.472826	0.715415	0.035739				
19.94498	9.13441	0.870563	0.050048				
19.14718	9.749521	1.03584	0.067459				
18.38129	10.32042	1.210113	0.088176				
17.64604	10.84926	1.39232	0.112378				
16.9402	11.33812	1.581458	0.140224				
16.26259	11.78896	1.776592	0.171854				
15.61209	12.20369	1.976839	0.207385				
14.9876	12.5841	2.181376	0.246922				
14.3881	12.93192	2.38943	0.29055				
13.81258	13.24881	2.60028	0.338338				
13.26007	13.53633	2.813251	0.390344				
12.72967	13.79601	3.027712	0.446609				
12.22048	14.02928	3.243078	0.507163				



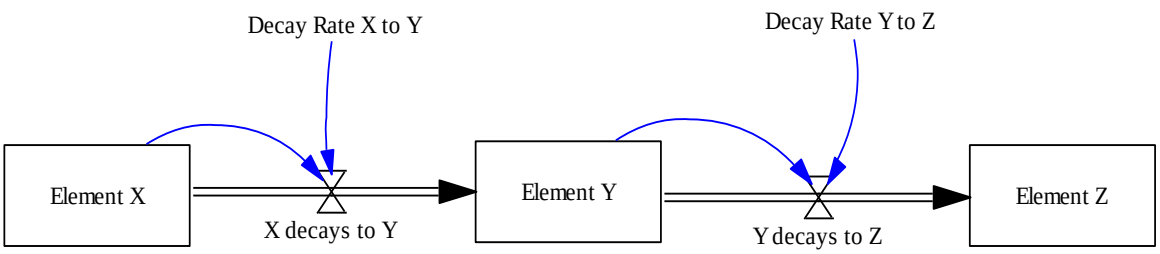
This model is no different from Radioactive Chain model: G decays to H, H decays to D, D decays to stable substance R.

Radioactive Chains

When a substance decays by radioactivity, it usually doesn't just transmute in one step into a stable substance; rather, the first substance decays into another radiative substance, which in turn decays into a third substance, and so on. For example, the uranium decay series is U238 -> Th 234 ->…. -> Pb 206, where Pb 206 is a stable isotope of lead. Suppose a radiative substance chains is described schematically by

$X \xrightarrow{-r} Y \xrightarrow{-s} Z$, where -r < 0, and -s < 0 are decay constants for substances X and Y, respectively, and Z is a stable substance.

We can draw the diagram of the radiative chain model as following:



Suppose x(t), y(t) and z(t) denote amounts of substances X, Y, and Z, respectively. The decay of element X is described by

$$\frac{dx}{dt} = -rx$$

Whereas the decay of element Y is

$$\frac{dy}{dt} = rx - sy$$

Since Y is gaining atoms from the decay of X and at the same time losing atoms because of its decay. Since Z is a stable element, it is simply gaining atoms from the decay of element Y:

$$\frac{dz}{dt} = -sy$$

In other words, a model of radioactive decay series for the three elements is a linear system of three first-order differentiations.

$$\frac{dx}{dt} = -rx$$
$$\frac{dy}{dt} = rx - sy$$
$$\frac{dz}{dt} = -sy$$

Looking at those equations, the system is close to popular SIR model.

Feedback:

The sequence of population models is to let students know they can create mathematics models base on their background. They can have a simple population model that can be applied to many real-life issues. Certainly they can refine to get better models. Usually, I used Have = Had + Change model at the first class of every course. The purpose is to let them know that they come to class to learn something they are not familiar. The learning of new skills is depending on their background. If they do not have good background, then they need to work extra hard to increase Change. Certainly, it also tells students that they cannot forget what they learned after final test. Everything they learned will be used in the future. Enzyne Dinamic Model is an example of application of Mathematics in Biology or Chemistry. No matter what topics in any subjects, as long as one can tell the story, one should be able to draw the diagram of the story and hence create a mathematics model. Cinderella Model is to let students know we can apply a fairy tale to STEM fields. I always believe that Mathematical modeling is the best way to connect classroom mathematics to real-world issues. I use those examples to encourage students to create mathematics model whenever they have a story.

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