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Control Charts, Scientific Derivation of Control Limits and Average Run Length

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Abstract: Control Charts, devised by W. A. Shewhart, in the 1950s have been greatly appreciated by Deming and Juran who introduced them in Japan. Good decisions depend on Scientific analysis of data. Data are collected, generally, in two ways: 1) one single sample of suitable size, 2) subsequent samples at regular intervals of time. Often, the data are considered Normally distributed; this is not completely right; data must be analysed according to their distribution: decisions are different with different distributions. We show the Theory to compute the Control Limits of the Control Charts and mention the case of a Professor of Statistics. We compare our findings with Shewhart findings; we extend the ideas to deal with "rare events" (data not Normally distributed); we compare our results, found by RIT, for cases in the literature: there is a big difference between the Shewhart Control Charts and the Time Between Events CC; considering that, future decisions of Decision Makers will be both sounder and cheaper, when data are not normally distributed. ARL depends on the data distribution, not only on the "false alarm rate". The novelty of the paper is due to the Scientific Way of Computing the Control Limits (that are an "Equivalence Classes"), both for the mean and for the variance.

Keywords: Control Charts, exponential distribution, JMP, Minitab, R, SAS, Reliability Integral Theory, TBE, T Charts

I. INTRODUCTION

The author knew about the problem of Individual Control Charts for Time Between Events (I-CC_TBE) in 1996(*Looking for Quality in "quality books"* [1]), by reading the Montgomery book[2], where he found the "example 6-6" (page 291, in the book). It is written there: *"A chemical engineer wants to set up a control chart for monitoring the occurrence of failures of an important valve. She has decided to use the number of hours between failures as the variable to monitor. … Clearly, time between failures is not normally distributed. [… constructing a time-between-events control chart is essentially equivalent to control charting an exponentially distributed variable.] Figure 6-23 is a control chart for individuals and a moving range control chart for the transformed times between failures … If a process change is made that improves the failure rate (such as a different type of maintenance), then we would expect to see the time between failures get longer. …"*. The data are the following (Table 1):

Table 1. Lifetime data (from Montgomery's book). *Notice that k=1 (sample size)*

816	729	4	143	431	8	2837	596	81	227	286	948	536	124	603	492	1199	1214	2831	96

The author agrees with Prof. Montgomery about his statement "*If a process change is made that improves the failure rate, then we would expect to see the time between failures get longer.*".This statement proves that Montgomery (and his lovers, suggesting his book to students) does not realise that, according to his definition, «quality gets worse, "*If a process change is made that improves the failure rate*", because "*the time between failures get longer.*"»: **Montgomery is in contradiction**. One can easily prove the mistake of Prof. Montgomery, if he/she reads and understands the Galetto books and papers: he had the chance to point out this to several professors, teaching Quality related matters: they did not understand. I asked to analyse scientifically the data: they were incapable, because they did not know the theory.

Montgomery finds the Process "In Control" (IC). Actually the Process "Out Of Control" (OOC).

The author, in his paper *Looking for Quality in "quality books"*[2], on 2001, showed the wrong analysis of Montgomery.

Since then, he has been informing the "quality" Community about that problem: he did not succeed.

Unfortunately, the ignorance of Peer Reviewers (PR) and of some Journal Editors helped the diffusion of wrong papers related to I-CC_TBE.

That's why, in this paper, we describe first the so-called Shewhart Control Charts (SCC), devised almost a century ago by W. A. Shewhart [3. 4] to control the production process of manufacturing products. In the 1950s, the Shewhart ideas have been greatly appreciated by Deming [5, 6] and Juran [7] who introduced them in

Japan. The success in Japan spurred the interest in the West… Since then, the use of SCC was termed Statistical Process Control (SPC).

The reader could ask himself: "Is there any need to discuss SPC, one century after its invention?"The answer is: YES! The reason of that is the diffused ignorance by many users (practitioners, scholars, Master Black Belts, Six Sigma instructors, students, professor, …), because there is not a scientific derivation of the formulae of Control Limits, LCL (Lower Control Limit) and UCL (Upper Control Limit).

The reader should be patient and read the paper if he wants to understand the big problem which Companies are confronting with.

People believe in ideas they need (and want) to believe: they believe that only papers published by reputed Journals are good papers (good=papers with Quality in them!).

They do not accept the evidence of wrong papers about I-CC_TBE, published in reputed Journals, and so the "Dis-quality" proliferates: they do not consider that *"the first step to Quality is recognising and accepting the actual problems* to be solved, without sweeping them under the carpet" (see the FAUSTA VIA).

The involved Journals with their PR and Editors are:*Quality and Reliability Engineering International*, *Quality Technology & Quantitative Management*, *Journal of Quality Technology*, *Reliability Engineering & System Safety*, *Quality Engineering*, *Journal of Statistics and Management Systems*, *Int. Journal for Research in Applied Science & Engineering Technology* and to the management of Minitab Inc.; neither the Editors nor the Minitabmanagement cannot acknowledge that!

The following "*XMAS Story*" shows the profound and pervasive ignorance about I-CC_TBE.

On December 18, 2022, we got the following request (verbatim, with «omissis» …):

Dear Professor,

*… I am … obtained my PhD in Statistical Process Monitoring … I am doing Post doctorate fellowship from … I am working on risk-based and percentile based control charts. I have seen your research interest is also in industrial statistics including quality control methods and Data Analysis. …. it will be a good opportunity for me to work under your kind guidance and have future collaboration. Waiting for your kind response.**Associate Professor of Statistics,** Department of Statistics, …*

FG accepted and suggested the I-CC_TBE problem…, sending him the data in Table 1 and asking if he could find the truth that the Process is OOC. In spite of the author's suggestions, after two weeks the Associate Professor of Statistics is still unable to see the problem. He acts the same way of the authors in the "ocean full of errors …" (see later), because he wrote papers with "misleading ideas". He is still using the same ideas as in the Excerpt 2 (see later).

Will the Associate Professor of Statistics see the truth?

End of "*XMAS Story*" (for now, when writing the paper). See the Conclusions…

A control chart is a plot of data (or of a statistic computed from data) over time, which is a desirable way to plot any set of data. The general idea is to identify common causes (of variation) and separate them from assignable causes, which are also called special causes, although the distinction between the two types of causes is often difficult to make.

A way of measuring the performance of CC is the expected length of time before a point plots outside the control limits. *If* the parameters were known (the case is inapplicable), the expected length of time before a point plots outside the control limits could be obtained as the reciprocal of the probability of a single point falling outside the limits when each point is plotted individually. This expected value is called the Average Run Length (ARL). It is desirable for the in-control ARL, indicated as ARL_0, to be reasonably large, so that false alarms will rarely occur (the probability of the case is α). With 3-sigma limits (see later), the ARL_0 is 1/.0027=370.37 *under the assumption of normality* and known parameter values. The ARL_0 being equal to the reciprocal of the sum of the tail areas results from the fact that the geometric distribution is the appropriate distribution, and the mean of a geometric random variable is $1/\alpha$.

Control Charts were devised by Shewhart using extensively the "*Normal Distribution*" of the data to be analysed to control the production process of manufacturing products: see several pages in the Shewhart (1931) [3] book and page 36 in the Shewhart (1936) [4] book. He does not connect (to our knowledge) explicitly the Control Limits to the Confidence Limits (of the Confidence Interval): on the contrary, we will do that and will show that many papers on control charts for TBE (Time Between Events) data provide wrong Control Limits, so making the users to take wrong decisions [see below the "ocean full of errors…"].

We think that this point is very important and surely a novelty in the literature on Control Charts.

We hope that any Peer Reviewer or any associate editor would not think that this work lacks sufficient novelty statistically in terms of theory and methods: please, read carefully the paper and do not think the following statements (as it was done by some authors (!) dealing with TBE data): "*We do not know this author and are not familiar with his work. His claim about our formulas being wrong is not justified by any facts or* *material evidence. <u>Our limits are calculated using standard mathematical statistical results/methods as is typical in the vast literature of similar papers</u>.*"

That is the dramatic problem that makes NO SENSE:

> *<u>limits are calculated using standard mathematical statistical results/methods as is typical in the vast literature of similar papers</u>*. (see the "*ocean…*")

TBE (Time Between Events) data are very much used in Healthcare settings. They are analysed by the use of I-CC_TBE, via the computation of the Control Limits: if the data plots within the Control Limits and there is no evident pattern, then the Process generating the data is considered In Control (IC).

Many papers and books compute wrongly the Control Limits and hence they find wrong ARL.

The situation is very dramatic; here, we cannot show all the cases.

We content ourselves to mention the papers we found in the web; see them in the "ocean full of errors …". It can be considered as a Literature Review.

We anticipate that the Process is OOC; see the Fig. 1 (as in*Looking for Quality in "quality books"* [1]).

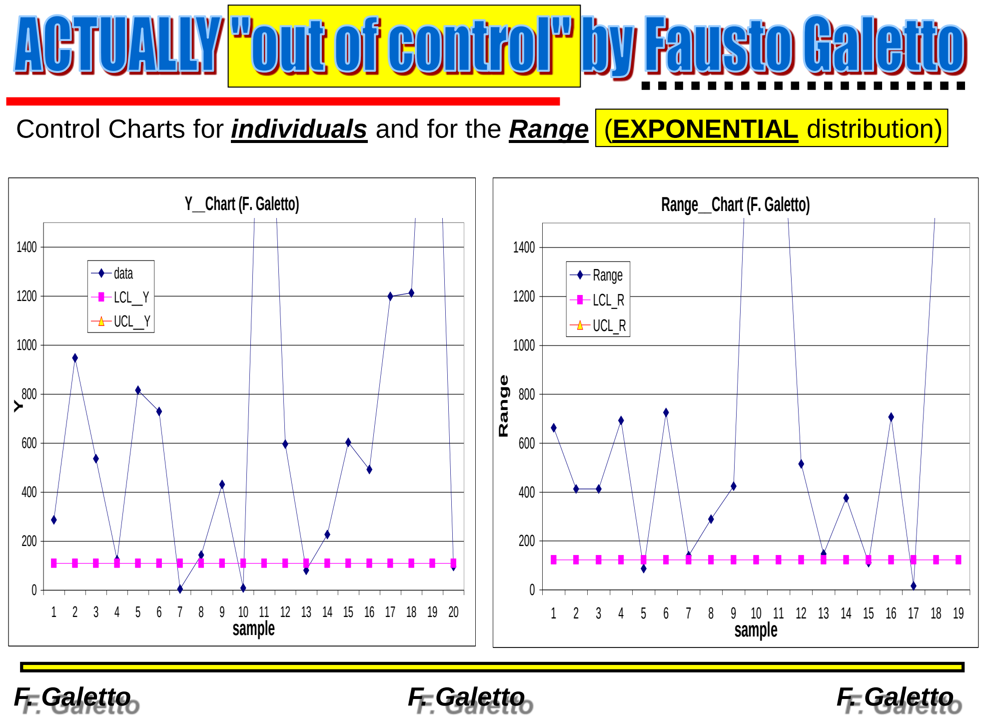


Figure 1. The Process (data in Table 1) is OOC both for the Individuals and for the Range; Scientific Control charts for valves [related to the data and control charts in Montgomery books].

This paper has the following structure: first, we introduce scientifically the concept of Confidence Interval (CI); secondly, we introduce scientifically the concept of Control Limits for the Control Charts and briefly present the Shewhart Control Charts and the Individual Control Charts (I-CC); thirdly, we show the correct control limits of control charts with exponentially distributed data, with the applications dealt in the "ocean full of errors …" and we will see the Minitab wrong calculations for the T Charts; we will show how RIT (Reliability Integral Theory) compute correctly the Control Limits for I-CC_TBE (I-CC for Time Between Events, exponentially distributed); fourthly, we show several wrong cases taken form the literature; finally, we show how to compute the ARL for I-CC. There is no specific "literature review" (but the "ocean full of errors…") because we are only interested in showing the RIT ability to solve correctly the *Control Charts for Exponentially Distributed Data*. RIT was devised by the author in 1975 (48 years ago) well before the T Charts invention. We, briefly, will present the RIT.

II. THE CONCEPT OF CONFIDENCE INTERVAL

If the reader thinks that it is <u>unwise</u> to dedicate a section to the concept of Confidence Interval (CI), we humbly ask him to stand back and meditate till the end of the section: after, he can disregard the content of it.

We think it is <u>*wise*</u> to do that, because the author met quite a lot of scholars (professionals, MBB, Professors, authors of papers, …) who did not know the correct concept. Moreover, we will see the scientific derivation of the formulae of <u>*Control Limits*</u>, Lower Control Limit (LCL) and Upper Control Limit (UCL) of

Control Charts: the Control Limits are strictly related to the ***Confidence Intervals***.

The concept of CI can be found in various books [7-13] (e.g. Juran 1988, Belz 1973, Rao 1965, Ryan 1989, Cramer 1961, Mood et al. 1963, Montgomery 1996, Galetto 1995-2000), but only the book [13] (Galetto 1995-2000) connects the CIs to the *Control Limits* of Control Charts.

The reader can find good ideas about the concept of CI (and many errors about it) in the paper [14].

Any Manager needs data to take decisions, suitable to the case he has to solve. But it is not enough: he needs also to analyse correctly the data and transform them into VALID information. To get this he NEEDS methods: better is if they are SCIENTIFICALLY sound. In the author's working life as Lecturer, Manager, Professor, papers writer, … he has been seeing a huge number of Lecturers, Managers, Practitioners, Professors, … taking wrong decisions BECAUSE they used wrong methods, NOT APPLICABLE to the problems they wanted to solve! This is the author's long experience in the Quality field, as teacher and Manager. When arguing on Scientific matters, everybody MUST act SCIENTIFICALLY. To fix better the ideas let's consider the following 10 data on 10 items: the first 5 data are the TIME TO FAILURE [failures occur at 115, 149, 185, 251, 350 (unit of measurement are not given)] and the other 5 are data on items that did not fail [NON_Failures at 350, 350, 350, 350, 350 (they are also named "suspended items"), according to a form of "***stopping rule***"]; the sets of such type of data are named "*INCOMPLETE samples*", because NOT ALL the data are "failures"; think to the data of survival of people to some drug cure: you do not wait until all die before taking decisions! ASSUMING that the distribution function $F(t)=1-\exp[-(t/\eta)^2]$ is the Weibull where the parameter is η (and the Mean is μ), and fixing CL=90% (Confidence Level), by making the right calculations we get that the Confidence Interval for the parameter μ (the mean) is 270.7--------582.8; GENERALLY the Statistics books do not consider the case of "INCOMPLETE samples"; they consider and provide only formulae for the "COMPLETE samples".

In the case of Control Charts we are very lucky because the samples are always "COMPLETE samples". Therefore, we consider here only those types of samples, in this paper. The definition of CI does not depend on the type of samples.

Before, we gave the Confidence Interval for the parameter μ [the mean of the Weibull distribution]: μ_L=270.7--------582.8=μ_U; CI is a ***numerical interval***, computed from the sampled data: length of the CI $d=\mu_U-\mu_L$ depends both on the Confidence Level (CL, 90%, in the example) and on the number of failures (not of the number of data). Therefore, to get the CI, we need

1. A set of k data, numbers (the sample, of sample size k, partitioned into "failures" and "suspended") collected from a test on a sample of physical items,
2. A CL, the Confidence Level, stated before any computation,
3. A *Probability* Distribution (depending on some ***parameters***, such as the "true" mean μ and the "true" variance σ^2) "generating" the collected data, from which we want to *estimate the parameters*,
4. A *Statistical* Distribution of the *estimators of the parameters* (which are Random Variables depending both on the RVs providing the data and on some function of the *parameters*),
5. A suitable formula to compute the CI from the k collected data.

The reader sees that there is a lot to be known if he wants to compute correctly the CI (and then, later, the Control Limits of the Control Charts).

Here we show how to do that *correctly* and we show the "Hidden Errors" one can make…

We assume, for the time being, that the*Probability* Distribution of the Random Variable X is the Normal Distribution (depending on the unknown *parameters*, the "true" mean μ and the "true" variance σ^2), given by the pdf (probability density function)

$$f_X(x;\ \mu,\sigma^2)=\frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\mu)^2/2\sigma^2} \tag{1}$$

We collect two samples of size k=5 each; we want to know if the two experimental means $\bar{x}_1$ and $\bar{x}_2$ (computed by the data x_{ij}, i=1, 2, j=1, 2, …5) suggest us that the "true" means μ_1 and $\mu_2$are to be considered different with CL=99.7%. The data are in the Table 2 (complete samples).

Table 2. Data and Confidence Intervals

Sample	datum	datum	datum	datum	datum	*mean*	CI_{Lower}	CI_{Upper}
1	809.4	758.4	633.9	820.6	530.9	*710.6*	*543.2*	*878.1*
2	767.2	968.4	595.3	482.5	659.9	*694.7*	*447.0*	*942.3*
pooled						*702.7*	*553.2*	*852.1*

From the Theory [7-15], we know that, with the Maximum Likelihood (ML) method, we estimate the "true" means μ_1 and $\mu_2$by the two experimental means $\bar{x}_1$ and $\bar{x}_2$ and the pooled mean $\bar{x}_{pooled}$ with the formulae, for complete samples

$$\bar{x}_i = \frac{\sum_1^5 x_{i,j}}{5} \quad i = 1,2 \;\; j = 1, \dots, 5 \quad and \quad \bar{x}_{pooled} = \frac{\sum_1^{10} x_{i,j}}{10} \tag{2}$$

and the "true" variances σ_1^2 and σ_2^2 by the two "corrected" experimental variances s_1^2 and s_2^2 and the pooled variance s_{pooled}^2 with the formulae

$$s_i^2 = \frac{\sum_1^5 [x_{i,j} - \bar{x}_i]^2}{4} \quad i = 1,2 \quad and \quad s_{pooled}^2 = \frac{4s_1^2 + 4s_2^2}{8} \tag{3}$$

Up to now, there is no innovation: only standard and classical known methods, for complete samples. IF, on the contrary, the samples were INCOMPLETE the formulae (2) and (3) ***would NOT*** be APPLICABLE!

The innovation arises when we compute the Confidence Intervals: the author knows only 3 books [11-13] (Cramer 1961, Mood et al. 1963, Galetto 1995-2000)that deal with CI as we are going to do now.

We consider the Cumulative Function N(x; μ, σ^2) of the Normal pdf (1), written for the Random Variable Mean$\bar{X}$

$$N\left(x;\, \mu, \sigma^2/n\right) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\sigma} e^{-(y-\mu)^2/(2\sigma^2/n)}\, dy = \int_{-\infty}^{(x-\mu)/(\sigma/\sqrt{n})} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz \tag{4}$$

and the parametric real function $g(x;\, \mu, \sigma) = (x - \mu)/\sigma$ which is a surface depending on μ and σ; putting $g(x;\, \mu, \sigma) = z$, we can draw the "level lines"

$$x = \mu - z\sigma \tag{5}$$

We consider the plane with axes μ (abscissa) and x (ordinate); we can draw the graph of the function x: it is a straight line parallel to the bisector, for any chosen values of z and of the parameter σ, as in figure 2; there is a double infinity of lines(depending on z and on σ).

Let's fix a Probability π=1-α, for the Normal Distribution of the Mean $\bar{X}$; choosing a symmetric interval, such as

$$\pi = 1 - \alpha = \int_{z_{\alpha/2}}^{z_{1-\alpha/2}} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz \tag{6}$$

we get two values $z_{\alpha/2}$ and $z_{1-\alpha/2}$ to be used in the formula (5): then, there is a single infinity of lines (depending on σ).

When we know the parameter σ, we have only two lines versus μ.

At any fixed value μ_0 (of the parameter μ), we find two points L and U (not random variables!) giving a Probability Interval for the Random Variable Mean $\bar{X}$, such that

$$P[L \le \bar{X} \le U] = \pi = 1 - \alpha \tag{7}$$

It is the vertical interval L-----U in figure 3.

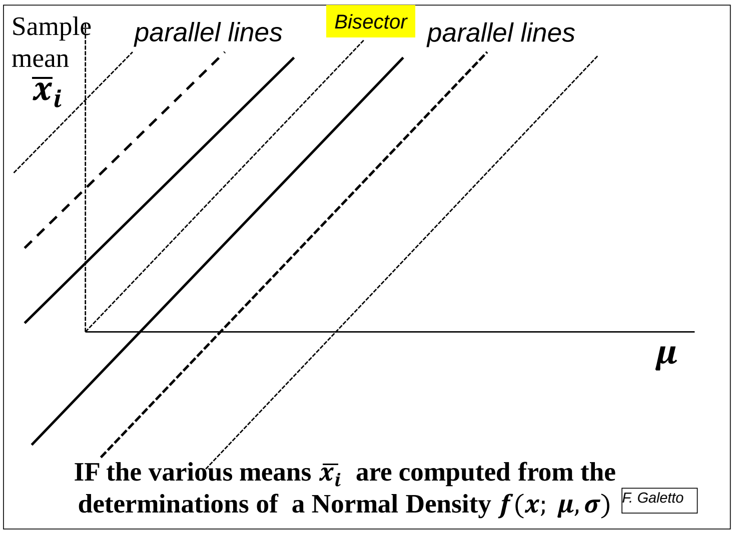


Figure 2. The "level lines" parallel to the bisector for the Normal Distribution

When we analyse the data of the i-th sample of size k (5, in table 1) we get a number, the experimental mean $\bar{x}_i$; the horizontal line from the point (0, $\bar{x}_i$), in figure 3, intersects the two lines (parallel to the bisector) in two points LLCi and ULCu giving the Confidence Interval LLCi--------ULCu.

Since the two lines are associated to the probability π=1-α, we say that the CI, LLCi--------ULCu, has a Confidence Level, CL=1-α: we are "confident" that the Confidence Interval LLCi--------ULCu could include the value μ_0 (*in the long-run*, a proprotion1-α of the CIs includes μ_0). NOTICE that ***the two intervals*** L-------------U and LLCi-------------ULCu ***are conceptually different***, even though they have the same length (caused by the Normal Distribution that is symmetric about the mean)!

This is of paramount importance for the Control Limits of the Control Charts.

This important point is neglected by all the authors mentioned in this paper… (“*ocean* …)

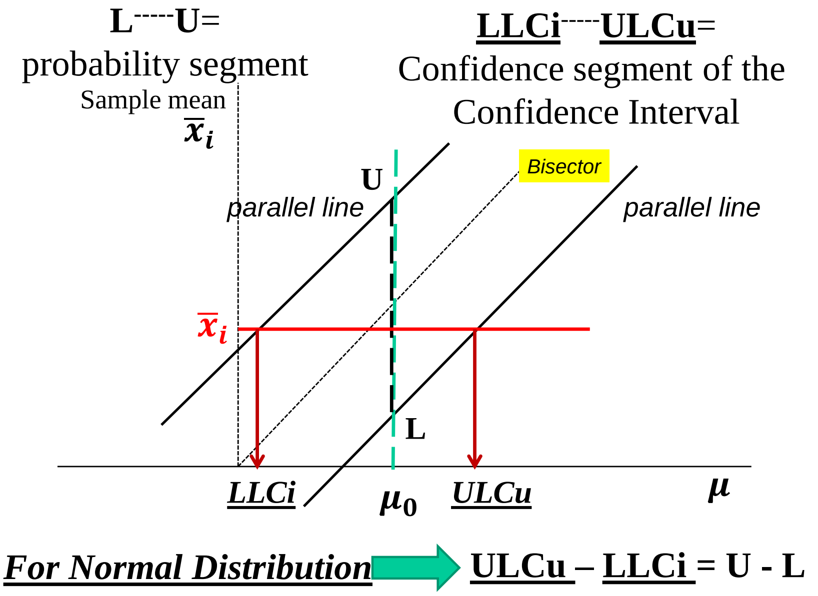


Figure 3. The TWO “level lines” parallel to the bisector for the Normal Distribution, providing the “Probability Interval” L--------U and the Confidence Interval LLCi--------ULCu

Taking into account the symmetry of the Normal pdf, we have, for the i-th sample,

$$L = \mu_0 - z_{1-\alpha/2}\sigma/\sqrt{k} U = \mu_0 + z_{1-\alpha/2}\sigma/\sqrt{k} \quad \text{and}$$
$$LLCi = \bar{x}_i - z_{1-\alpha/2}\sigma/\sqrt{k} ULCu = \bar{x}_i + z_{1-\alpha/2}\sigma/\sqrt{k} \quad (8)$$

Looking at the formulae (8), it is ***very*** ***tempting*** *to draw the WRONG conclusion that we get the Confidence Interval by putting the experimental mean*$\bar{x}_i$*in place of* the hypothesed value μ_0 of the parameter μ.

This is the error that many people make when they consider distributions different from the normal one (or symmetric pdf).

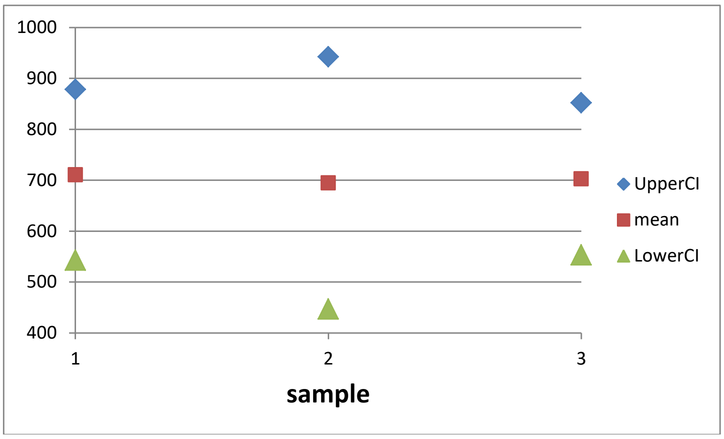


Figure 4. Confidence Intervals of the samples in Table 1 (3=pooled sample)

One could use the same ideas, given before, but not the same formulae, for finding the Confidence Interval of the variance σ^2 or of the standard deviation σ, or of any other quantities such as (e.g.) the p-values.

The “lines” to be used for the variance σ^2 are *not parallel* to the bisector!

Formulae (8) are valid when the standard deviation σ is known. If the standard deviation is estimated from the collected data (normal), then the quantity z computed from the normal distribution must be substituted by t taken from the t (Student) distribution: the values $t_{1-\alpha/2}$ must be used: $LLCi = \bar{x}_i - t_{1-\alpha/2}s/\sqrt{k},\ ULCu =$

$\bar{x}_i + t_{1-\alpha/2} s/\sqrt{k}$.

The CIs can be used for comparing the mean of each sample versus the means of the other sample, to assess either if they are "*statistically* different" or if they are "*statistically* equivalent".

See the figure 4. We see that each *experimental mean* (red square) of any sample is comprised in the Confidence Interval of the other sample: that means that the "true" means μ_1 and μ_2, estimated by the *experimental means*, are "*statistically* equivalent". Any CI is the set of all the numbers "*statistically* equivalent" between them: CI is an "*equivalence class*"!

III. THE CONCEPT OF CONTROL LIMITS FOR THE CONTROL CHARTS

We use here the ideas of the previous section to find the Control Limits for the Control Charts. Control Charts are the tool for assessing the "health" of the process: it is the "*thermometer for measuring the fever of a Process*".

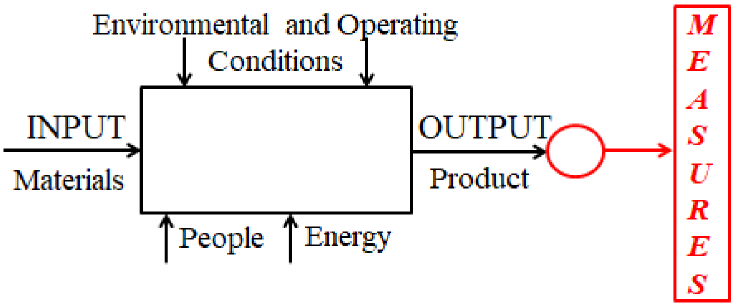
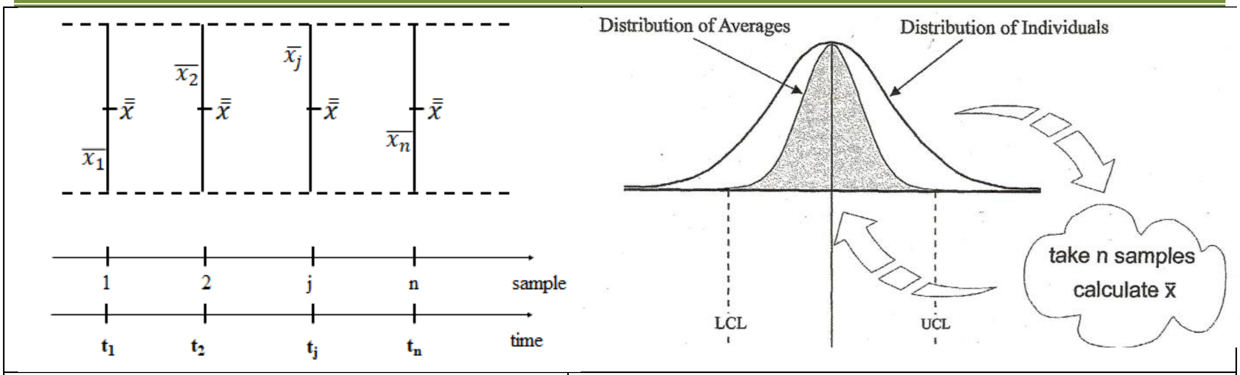


Figure 5. Schematic diagram of a process and its output (products and measures)

Control Charts are a statistical tool for monitoring the "measurable output" of a Process (Production or Service Process).The "measurable output" of the Process is provided by the products produced by the process (fig. 5): we measure a "quality characteristic" of the product [a characteristic whose measure states if the "measured" product has quality (good health) or if it is defective (fever)]. The "measurable output" of the Process can be viewed as a "Stochastic Process X(t)", ruled by a probability density for any set of n "Random Variables RV" $X(t_1)$, $X(t_2)$, …, $X(T_N)$, considered at the "time instants" t_1, t_2, …, T_N, of the Stochastic Process X(t)". X(t) can be multidimensional or unidimensional: generally, in applications, there is a single measured quality characteristic X(t); such control charts (CC) are routinely called univariate SPC charts in the literature.

In many applications, generally, the data plotted (on the Control Chart) are the means $\bar{x}(t_i)$, determinations of the Random Variables $\bar{X}(t_i)$, i=1, 2, ..., n (n=number of the samples) computed from the collected data x_{ij}, j=1, 2, ..., k (k=sample size); x_{ij} are the determinations of the RVs $X(t_{ij})$ at *very close instants* t_{ij}, j=1, 2, ..., k; the RVs (random variables) $\bar{X}(t_i)$ are assumed to follow a normal distribution *because* (Central Limit Theorem) they are the means of samples with sample size, k, each; usually k=5 [they were the original ideas of W. Shewhart]. For each RV $\bar{X}(t_i)$, mean of the process (at time t_i), the mean of RVs $X(t_{ij})$ j=1, 2, ..., k, (k data sampled, at very near times t_{ij}); we assume here that it is distributed as $\bar{X}(t_i) \sim N(\mu_{\bar{X}(t_i)}, \sigma^2_{\bar{X}(t_i)})$ with mean $\mu_{\bar{X}(t_i)}$ and variance $\sigma^2_{\bar{X}(t_i)}$: this is the assumption of W. A. Shewhart on page 278 of his book [3] (Shewhart 1931), and justified on page 289 by writing "*… we saw that, no matter what the nature of the distribution function of the quality is, the distribution of the arithmetic mean approaches normality rapidly with increase in n* (notice that *his n* is our k)*, and in all cases the expected value of means of samples of n* (our k) *is the same as the expected value of the universe*"; here we accept this assumption for future comparisons. Another common assumption for Variable Control Charts is that the RVs $X(t_{ij})$ are also independents and we can compute a grand mean $\bar{\bar{X}}$ [mean of all the RVs $X(t_{ij})$] that is distributed as $\bar{\bar{X}} \sim N(\mu_{\bar{\bar{X}}}, \sigma^2_{\bar{\bar{X}}})$.



$\bar{x}(t_i)$of the process and the "grand mean" $\bar{\bar{x}}$	Individuals and Means (Averages)

Figure 6. The Individuals, the "means" $\bar{x}_i$of the process and the "grand mean" $\bar{\bar{x}}$

In any Production or Service process (figure 5 and 6), modelled by the "Stochastic Process" X(t), there is a "background noise (inherent natural variability)", which makes the process to provide a variable output: a certain amount of inherent natural variability always exists in any process output (it is named "due to chance causes of variability"; W. A. Shewhart terms it ″*constant systems of causes*″); such a process is said to be ″statistically In Control″, IC (*no fever*): in such a case all the means $\mu_{\bar{X}(t_i)}$, estimated by the experimental means $\bar{x}_i = \bar{x}(t_i)$, have to be considered "equivalent (i.e. equal, $\mu_{\bar{X}(t_q)} = \mu_{\bar{X}(t_r)}$for any q≠r)" between them and to $\mu_{\bar{\bar{X}}}$.

If a product (output of the process) has variability, in its quality characteristics, greater than the inherent natural variability we say that the process is an ″Out-Of-Control process″ [OOC (the process has "*fever*")] and operating in the presence of″*assignable causes of variation*″: in such a case some of the means $\mu_{\bar{X}(t_i)}$, estimated by the experimental means $\bar{x}_i = \bar{x}(t_i)$, have to be considered "NOT-equivalent (i.e. statistically different)".The Control Charts are a tool used to understand if a process is IC or OOC [1-15].

We consider here only Variable Control Charts (see Fig.6), used when the quality characteristics of the output are measured: it is a graphical display of a quality characteristic that has been measured or computed from the data of several samples versus the sample number or time. It is made four elements: the data plotted [determinations of the "Stochastic Process" X(t)] and 3 lines, a centre CL, a lower line LCL (Lower Control Limit) and an upper line UCL (Upper Control Limit). If a point plots outside of the control limits (or there are significant patterns) then we interpret it as evidence that the process is Out Of Control (OOC): investigation is needed to understand and find the "assignable causes of variation". The Control Limits are determined in such a way that if the process output has only chance (random) variability, that is, it is In Control (IC), then the data plotted in the control chart are 99.7% between LCL and UCL (and there are no significant patterns). This is the Theory of W. Shewhart, devised almost a century ago, in the 1920s, at Bell Telephone laboratories.

In Fig.6 the distribution (on the right), the determinations of the RVs $\bar{X}(t_i)$ and $\bar{\bar{X}}$ are shown (left).

So far we used known ideas.

Let's now use the concepts of the section entitled "The concept of Confidence Interval".

Let's consider first the Confidence Interval of the unknown mean μ of the process.

We act in this way:

(i) We fix the probability π=1-α=0.9973
(ii) We collect n (usually n>20) samples, of size k each (usually k=5), i. e. N > 100 x_{ij} measures (N=nk),
(iii) We compute the "*experimental mean*" $\bar{x_i}$ of the each i-th sample [determination of the RV $\bar{X}(t_i)$] and the "*experimental standard deviation s_i*" of the i-th sample,
(iv) We compute the "*experimental grand mean*" $\bar{\bar{x}}$ of all the n samples [determination of the RV $\bar{\bar{X}}$] and the "*experimental standard deviation s*",
(v) We draw the two lines parallel to the bisector with the "*experimental standard deviation s*" and we find the value $t_{1-\alpha/2}$ related to the "*probability interval* L-----U",
(vi) We draw the horizontal line at value $\bar{\bar{x}}$ (see figure 3 with $\bar{\bar{x}}$ in place of $\bar{x_i}$ and N in place of n) intersecting the two lines parallel to the bisector and we find the "*Confidence Interval* $^{(N)}$LLCi-------- $^{(N)}$ULCu" of the unknown mean μ of the process [here, the index $^{(N)}$ states that the CI is computed with the N data],
(vii) NOTICE that the last computations come from the total N data.

The "*Confidence Interval* (N)LLCi--------(N)ULCu" of the unknown mean μ of the process is not suitable to compare each mean $\mu_{\bar{X}(t_i)}$, estimated by the "*experimental mean*" $\bar{x}_i$, to$\mu_{\bar{\bar{X}}}$, estimated by the grand mean $\bar{\bar{x}}$, and to all the other means $\mu_{\bar{X}(t_i)}$, i=1, 2, …, n, estimated by their "*experimental means*": if the Process is "In Control" the variation of each one the means $\mu_{\bar{X}(t_i)}$ is "stable", i.e. we have to use the "*experimental variability* $s/\sqrt{k}$" (related to $\mu_{\bar{\bar{X}}}$) and not $s/\sqrt{N}$ (as done before).

Doing that we have the formulae $LLCi = \bar{\bar{x}} - t_{1-\alpha/2}s/\sqrt{k}$ and $ULCu = \bar{\bar{x}} + t_{1-\alpha/2}s/\sqrt{k}$, which provide the Control Limits of the Control Chart (LCL and UCL)

$$LCL = \bar{\bar{x}} - t_{1-\alpha/2}s/\sqrt{k} \quad \text{and} \quad UCL = \bar{\bar{x}} + t_{1-\alpha/2}s/\sqrt{k} \tag{9}$$

The formulae (9) allow us to compare each mean $\mu_{\bar{X}(t_q)}$, q=1,2, …, n, to any other mean $\mu_{\bar{X}(t_r)}$r=1,2, …, n,and to the grand mean $\mu_{\bar{\bar{X}}} = \mu$; hence, so doing, we assess if the process is IC or OOC (see the figure 6, where the process is IC; consider also the figure 3).

Generally, in the Control Chart use, we set 3=$t_{1-\alpha/2}$, both for the Control Chart of the mean μ and of the standard deviation σ (even though the distribution of the "variability σ" is far from being normal!).

$$\bar{\bar{X}} \pm 3\frac{\sigma}{\sqrt{n}} = 13{,}540 \pm 3\frac{440}{\sqrt{20}} = \begin{cases} 13{,}245 \\ 13{,}835, \end{cases}$$

and

$$\bar{\sigma} \pm 3\frac{\sigma}{\sqrt{2n}} = 423 \pm 3\frac{440}{\sqrt{40}} = \begin{cases} 214 \\ 632. \end{cases}$$

Excerpt 1. From Shewhart book [3]

See, in the excerpt 1, what W. A. Shewhart [3] (1931) wrote on page 294 of his book, where $\bar{X}$ stands for our $\bar{\bar{x}}$ and σ stands for our s; $\bar{\sigma}$ is the "estimated mean standard deviation if all the s_i" and n stands for our sample size k.

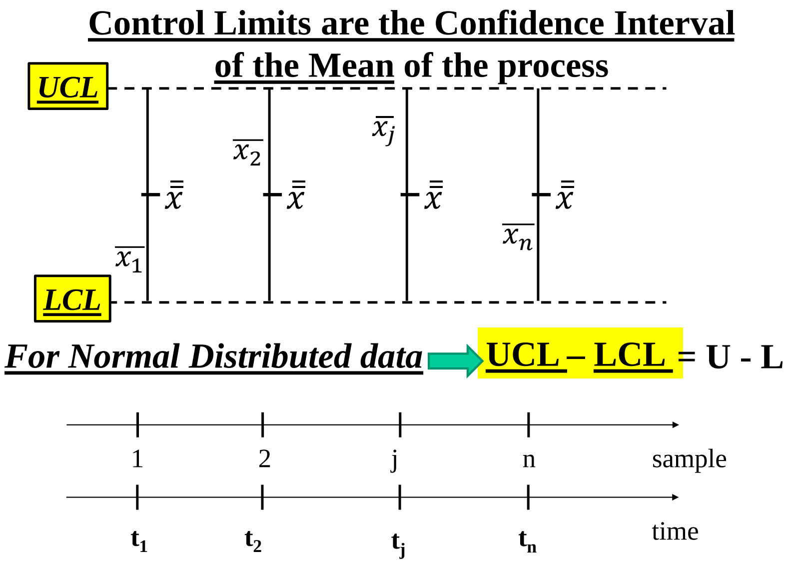


Figure 7. Control Limits LCL$_X$----UCL$_X$=L----U (Probability interval), for Normal data

Calling X the "quality characteristic" that measures the quality of the product, produced by the production process, it is customary to write the following formulae for the Control Charts (the coefficient A_2 depends on the sample size k):

$$LCL_X = \bar{\bar{x}} - A_2\bar{R}, \qquad CL_X = \bar{\bar{x}}, \qquad UCL_X = \bar{\bar{x}} + A_2\bar{R} \tag{10}$$

The interval LCL$_X$------UCL$_X$ (Control Limits, on the horizontal axis, in figure 2 and on the vertical axis, in figure 5) is the "Confidence Interval" with "Confidence Level" 1-α=0.9973 for the unknown mean $\mu_{X(t)}$ of the Stochastic Process X(t).

A similar control chart is drawn for the range by making a "**bigger***mental leap*" [the distribution of $\bar{R}$ is not normal!] (the coefficient D_3 and D_4 depend on the sample size k)

$$LCL_R = D_3\bar{R}, \qquad CL_R = \bar{R}, \qquad UCL_R = D_4\bar{R} \tag{11}$$

The interval LCL_R-----------UCL_R is the "Confidence Interval" with "Confidence Level" CL=1-α=0.9973 for the unknown Range of the Stochastic Process X(t).

Notice that the Control Interval (Confidence Interval) $\mathbf{UCL_X\text{-}LCL_X=U\text{-}L}$ (Probability Interval) for normally distributed data and that LCL_X can be obtained from L by substituting μ with $\bar{\bar{x}}$; the same for UCL_X and U.

It is customary to use both the formulae (10) and (11) also for NON_normal data: in such a case the NON_normal data are transformed in order to "produce Normal data" and to apply formulae (10) and (11).

Sometimes we have few data and then we use the so called "*individual control charts*" I-CC. The "*individual control charts*" I-CC are very much used for exponentially distributed data: they are named "rare events Control Charts for TBE (Time Between Events) data", I-CC_TBE.

There are scholars in the "ocean full of errors …" (see the next section) who think and say that

- "*The problem of monitoring TBE that follow an exponential distribution is well-defined and solved*. I do not agree that "nobody could solve scientifically the cases"**"**.
- "*We do not know this author and are not familiar with his work. His claim about our formulas being wrong is not justified by any facts or material evidence. Our limits are calculated using standard mathematical statistical results/methods as is typical in the vast literature of similar papers*."

To show the big problem (see "ocean full of errors…") about the I-CC for TBE we use the data in table 1; it shows the data (named *lifetime*) of a case, found as Example 7.6 in the Montgomery (1996) book[2]. The data are (exponentially distributed).

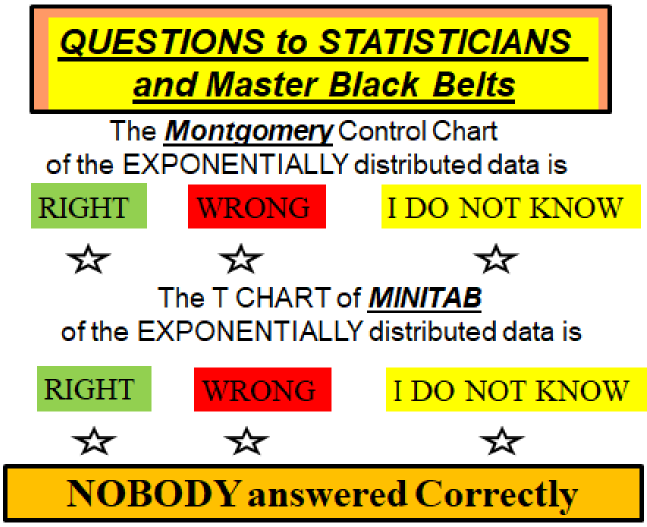


Figure 8. Question asked to MBB, Statisticians, Professors of Statistics and Experts

On December 2019 the author asked a question in a post at the site iSixSigma [14]: https://www.isixsigma.com/topic /control-charts-non-normal-distribution related to control charts by saying "*I would like to get solution to the cases shown in the file. THANKS in advance. Fausto Galetto* (*with the attachment: ISIXSIGMA-INSIGHTS_Two cases for Master- Black-Belts-dec-2019.docx*". The cases were related to control charts where the data are exponentially distributed. The first of the cases was taken from the book "Introduction to Statistical Quality Control", Wiley & Sons 1996 (D. C. Montgomery) [table 1].

The author knew about that since 1996; Montgomery dealt with it (in the same way) in all the later editions of the book. The iSixSigma "experts" (see figure 8) were unable to provide a correct way to solve the cases and did not wanted to accept that the Montgomery's solution was doubtful because *he finds that the process is In Control (IC),* while actually the process is Out Of Control (OOC); at least 70 Master Black Belts, of several countries (Italians as well), and more than 60 "experts" (statisticians, professionals, engineers, …) have been asked to find the solution (see figure 7) of the two cases of Control Charts, in three "social" groups: iSixSigma, Academia.edu and Research Gate. Up to now, January 2023, nobody provided any good solution to the problem!

Any scholar who want to learn Control Charts both with normal distribution and TBE (exponential and Weibull distribution) can usefully read the book “Statistical Process Management” [15] (Galetto 2019).

Remember the *"XMAS Story"*…

IV. INDIVIDUAL CONTROL CHARTS (I-CC) AND EXPONENTIALLY DISTRIBUTED DATA. PART 1

The I-CC are very similar to the Control Charts (with k=5 sample size); the only difference is that the sample size is k=1 for I-CC; see figure 9.

The “grand mean” $\bar{\bar{x}}$, in this case, becomes the mean $\bar{x}$.

Since we cannot compute the standard deviation of each sample, to compute the Control Limits (LCL and UCL) we are forced to use the differences of each datum x_i to the following datum x_{i+1} for i=1, …, n-1 (n=total number of data).With the differences $x_{i+1} - x_i$ we can compute the n-1 ranges and then we can use the formulae (10) and (11), for the Normal distributed data.

As said already, if the data are not Normally distributed it is wrong to use the formulae (10) and (11).

What then are forced to do the scholar who do not have the right Theory?

They transform the data in order to have the “transformed data” Normally distributed.

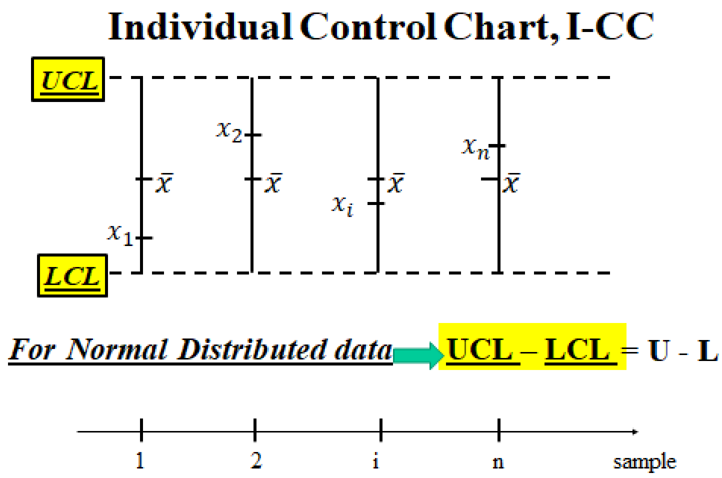


Figure 9. Individual Control Chart. *Notice that k=1 (sample size)*

D. C. Montgomery [2] did that: he transformed the Exponential data into Weibull data with shape parameter β=1/3.6, as suggested by Nelson.

He did not care if the transformation would give sound analysis: he used it blindly.

Before using a transformation, any scholar should see if it is suitable, because, as said by Deming [5-6] (1986, 1997),"*Management need to grow-up their knowledge because experience alone, without theory, teaches nothing what to do to make Quality*" and "*The result is that hundreds of people are learning what is wrong. I make this statement on the basis of experience, seeing every day the devastating effects of incompetent teaching and faulty applications.*" and, moreover, "*It is necessary to* ***understand the theory*** *of what one wishes to do or to make.*" [Deming (1986)]

Let be y_i the original data (exponential, in table 1) [whose “true” Control Charts you see in Fig. 1] and $x_i=y_i^{1/3.6}$ the transformed (Weibull) data; Montgomery uses a I-MR Chart where in the upper graph the individual x_i are plotted with their mean $\bar{x}$ and control limits and in the lower graph the individual moving ranges $MR_i=|x_i - x_{i+1}|$ are plotted with their mean $\overline{MR}$ and control limits [Fig.10 is the same graph of Montgomery’s book].

According to the figure 10, Montgomery writes “*Note that the control charts indicate* a state of control, *implying that the failure mechanism for this valve is constant. If a process change is made that improves the failure rate (such as a different type of maintenance action), then we would expect to see the mean time between failures get longer. This would result in points plotting above the upper control limit on the individuals control chart*”.

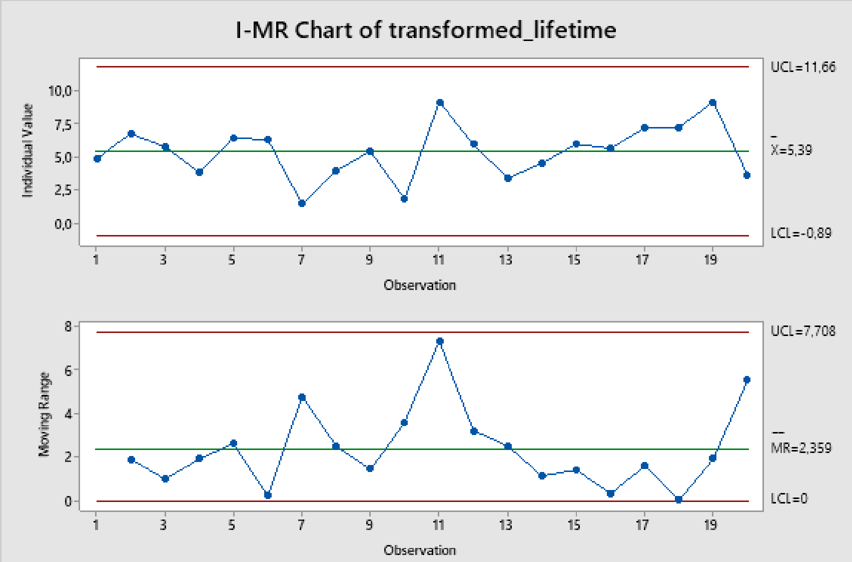


Figure 10. Individual and Moving Range chart of *"transformed"* Montgomery data (as suggested by Nelson). Minitab 19&20&21 used (F. Galetto).

Is this a true picture of the process? Perhaps this "In Control" depends on the formulae used!

Using other software, either R, or SixPack or JMP or SAS, or SPSS we would have the same picture of the process (for the transformed data).

If one wants to use the (*inapplicable*) formulae (10) and (11), for individual exponential data, he finds Fig. 11:

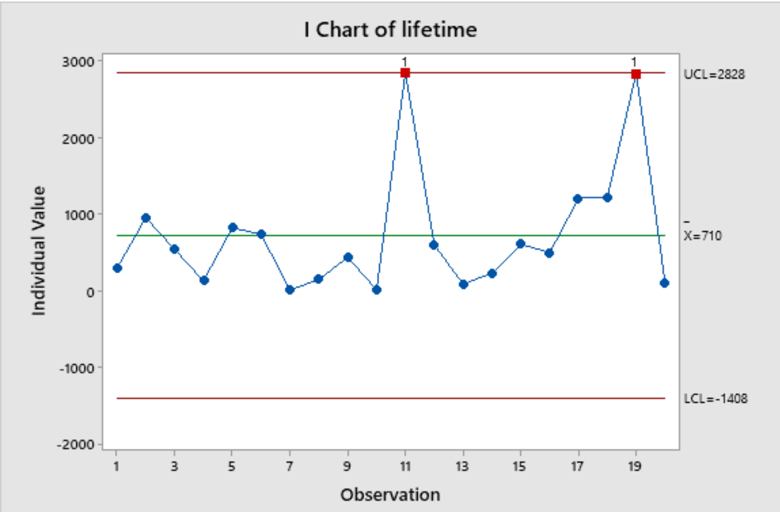


Figure 11. Individual chart of Montgomery data. Minitab 19&20&21 used (assuming…) [**Test Results for I Chart of lifetime** TEST 1. One point more than 3,00 standard deviations from center line. Test Failed at points: 11; 19]

Then *we have two contradictory conclusions*! See both the Fig.10 and 11…

The *discussants*, in iSixSigma (https://www.isixsigma.com), suggested using the Minitab Software and its "*T Charts*", assuming that the T Charts are the good method to deal with "rare events" (Galetto 2019, 2020).

Now, see the Minitab T Chart (figure 11; it is figure 7a in Galetto [16)] 2020). The process is "In Control", again.

Is this a true picture of the process?

NO. Perhaps this "In Control" depends on the formulae used!

Actually, the process is Out Of Control [Fig. 1]!

After these findings, the author tried to inform the "Scientific Community" about the problems of I-CC_TBE: wrong Control Limits in them.

At that time, several documents were analysed [looking for them in the Web…(***notice*** that one of the authors is very well known; he has *6969 Citations*! Does that mean that he wrote good papers? Absolutely not!)]; see the following "*ocean full of errors by …*":

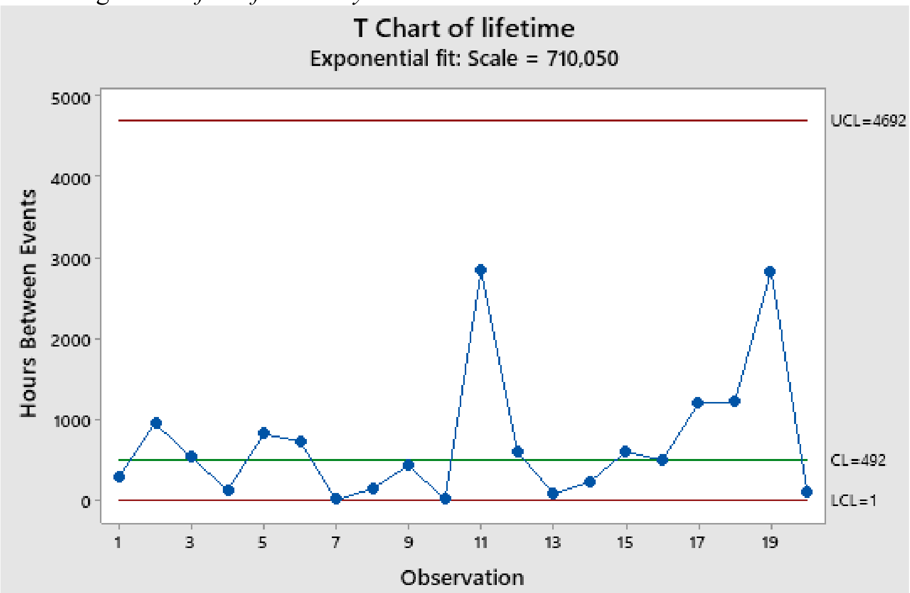


Figure 12. T Chart of Montgomery data. Minitab 19&20 used (F. Galetto). The same figure can be found using the new release Minitab 21 [the problem is still present]

The "ocean full of errors by…"
Dovoedo Y. H. and Chakraborti S., "Boxplot-based Phase I Control Charts for Time Between Events",*Quality and Reliability Engineering International*, N. Kumar, A. C. Rakitzis, S. Chakraborti, T. Singh (2022), "Statistical design of ATS-unbiased charts with runs rules for monitoring exponential time between events", *Communications in Statistics - Theory and Methods*, Jones LA, Champ CW., "Phase I control charts for times between events", *Quality and Reliability Engineering International*, Fang Y. Y., Khoo M. B., Lee M. H., "Synthetic-Type Control Charts for Time-Between-Events Monitoring", *PLoS ONE*, N. Kumar, S. Chakraborti, "Improved Phase I Control Charts for Monitoring Times Between Events" [*Quality and Reliability Engineering International*, 2014] (*found online, 2021, March*), Dovoedo Y. H., "Contribution to outlier detection methods: Some Theory and Applications", (*found online, 2021, March*), Saghir, Lin, Abbasib, Ahmada, "The Use of Probability Limits of COM–Poisson Charts and their Applications", (*found online, 2022, December*), Liu J., Xie M., Sharma P., "A Comparative Study of Exponential Time Between Event Charts", *Quality Technology & Quantitative Management*, Frisén, M., "Properties and Use of the Shewhart Method and Followers", *Sequential Analysis*, Woodall, W. H., "Controversies and Contradictions in Statistical Process Control", *Journal of Quality Technology*, Kittlitz, R. G., "Transforming the exponential for SPC applications". *Journal of Quality Technology*, Schilling, E. G., Nelson, P. R. "The effect of non-normality on the control limits of X charts", *Journal of Quality Technology*, Woodall, W. H., "The use of control charts in health-care and public health surveillance", *Journal of Quality Technology*, Xie, M., Goh, T. N., Kuralmani, V., "Statistical Models and Control Charts for High-Quality Processes", (Boston, MA: *Kluwer Academic Publisher*, 2002), Xie, M., Goh, T. N., Kuralmani, V., "Chapter 3 of the book Statistical Models and Control Charts for High-Quality Processes" (Boston, MA: *Kluwer Academic Publisher*, 2002), Xie, M., Goh, T. N., Ranjan, P., "Some effective control chart procedures for reliability monitoring", *Reliability Engineering & System Safety*, Xie, M., "Some Statistical Models for the Monitoring of High-Quality Processes", Boston, chapter 16 in the book *Engineering Statistics (Pham Editor): Springer-Verlag*, Zhang, C. W., Xie, M., Goh, T. N. "Economic design of exponential charts for time between events monitoring", *International Journal of Production Research*, Zhang, C. W., Xie, M., Goh, T. N., "Design of exponential control charts using a sequential sampling scheme", *IIE Transactions*, Zhang, H. Y., Xie, M., Goh, T. N., Shamsuzzaman, M. "Economic design of time-between-events control chart system", *Computers and Industrial Engineering*, E. Santiago, J. Smith, "Control charts based on the Exponential Distribution", *Quality Engineering*, Nasrullah Khan & Muhammad Aslam, "Design of an EWMA adaptive control chart using MDS sampling", *Journal of Statistics and Management Systems*, S. Balamurali& Muhammad Aslam, "Variable

batch-size attribute control chart", *Journal of Statistics and Management Systems*.
On September 2022, the author looked (in the web) for other TBE (Time Between Event) papers and books to see their way of dealing with "Rare Events" Control Charts; he copied 77 pages of documents (several from Consultants); he downloaded 32 papers (Open Source). Several Journals asked from 15 $ to 60 $, to download a paper. The Open Source are:
"Control Chart: Charts for monitoring and adjusting industrial processes", "TOOL #6 - XBar& R Charts", "Integrating Quality Control Charts with Maintenance", "A Brief Literature Review", "Paper SAS4040-2016, "Improving Health Care Quality with the RAREEVENTS Procedure Bucky Ransdell, SAS Institute Inc.", "Performance Criteria for Evaluation of Control Chart for Phase II Monitoring", "(Thesis) A Comparative Study of Control Charts for Monitoring Rare Events in Health Systems Using Monte Carlo Simulation", "A study on the application of control chart in healthcare", "Control Charts for Monitoring the Reliability of Multi-State Systems", "Part 7: Variables Control Charts2, "A Control Chart for Gamma Distribution using Multiple Dependent State Sampling", "A Variable Control Chart under the Truncated Life Test for a Weibull Distribution", "Plotting basic control charts: tutorial notes for healthcare practitioners", "Appendix 1: Control Charts for Variables Data – classical Shewhart control chart", TRUNCATED ZERO INFLATED BINOMIAL CONTROL CHART FOR MONITORING RARE HEALTH EVENTS", "Comparison of control charts for monitoring clinical performance using binary data", "A number-between-events control chart for monitoring finite horizon production processes", "Rare event research: is it worth it?", "Quality Improvement Charts: An implementation of statistical process control charts for R", "Control Chart Overview", "Statistical Process Control Monitoring Quality in Healthcare", "A Control Chart for Exponentially Distributed Characteristics Using Modified Multiple Dependent State Sampling", "Synthetic-Type Control Charts for Time-Between-Events Monitoring", "A systematic study on time between events control charts", "Lifestyle Managementthrough System Analysis Monitor Progress", "Multivariate Time-Between-Events Monitoring – An overview and some (overlooked) underlying complexities", "A Comparison of Shewhart-Type Time-Between-Events Control Charts Based on the Renewal Process", "Control Charts for Monitoring Time-Between-Events-and-Amplitude Data", "How to Measure Customer Satisfaction Seven metrics you need to use in your research".

The "ocean full of errors by …"

All the papers in the above ***"****ocean full of errors by …****"***have the same problem: wrong Control Limits; the authors confound the concepts, by stating that LCL and UCL (that actually are the Confidence Limit!) are the limits L and U of the Probability Interval.

They were (and are) "*lead into temptation and delivered into evil*" by the (*inapplicable*) formulae (10) and (11), that are applicable only for the normal data and ***not*** for exponential data, in spite of the wrong statements (excerpt 2) below about "*limits… typical in the vast literature…*" and "*The problem … exponential distribution is well-defined and solved*".

They use wrongly the Probability Interval L------U as though it were the Control Interval LCL------UCL and put the estimates, either $\bar{t}$ or $1/\bar{t}$, of the parameters in place of the parameters, θ or $\lambda=1/\theta$. See the excerpt 2: "the *temptation* and the *evil*". Notice that an author in the "ocean…" is Associated Editor of …

Now, the author is at risk. IF the Peer Reviewers (PRs) are taken from the ***"****ocean …****"*** they will not acknowledge the errors and could think:

- "*The problem of monitoring TBE that follow an exponential distribution is well-defined and solved*. I do not agree that "nobody could solve scientifically the cases"**"**.
- "*We do not know this author and are not familiar with his work. His claim about our formulas being wrong is not justified by any facts or material evidence. Our limits are calculated using standard mathematical statistical results/methods as is typical in the vast literature of similar papers.*"

and, hence, the paper would get the following evaluation:

- "*Your manuscript is unsuitable for publication in …. For your information I have attached comments at the bottom of this email. I hope you will find them to be constructive and helpful. You are of course now free to submit the paper elsewhere should you choose to do so. Thank you for considering …. for the publication of your research. I hope the outcome of this specific submission will not discourage you from the submission of future manuscripts.*"
- "*The* ***associate editor***(notice!) *has stated that the work lacks sufficient novelty statistically in terms of theory and methods.*"

Therefore,

- the readers **can** know the <u>wrong methods</u> that have "*<u>sufficient novelty statistically in terms of theory and methods</u>*." (see the *"ocean …"*)
- but, on the contrary, they **cannot** know the ***<u>rigth</u>*** <u>methods</u> that prove how many incompetent published wrong papers, that diverted, are diverting and will divert people from learning ***<u>scientific</u>*** <u>methods</u>.

Typical statement by ALL …

A uniform model the exponential TBE charts is that the occurrence of events is modelled by a Poisson process, and the time between events X_i (i=1, 2, …) re independent and identically distributed random variables with pdf $f(x) = \theta^{-1}\exp(-x/\theta)\, for\ x \geq 0$, 0 otherwise, where θ is the "mean time between events".
The Control Chart plots the quantity produced before observing an event;
The Control Limits can be calculated as

$$LCL = \theta\ln(1-\alpha/2), \qquad UCL = \theta\ln(\alpha/2)$$

Liu J., Xie M., Sharma P., "A Comparative Study of Exponential Time Between Event Charts", *Quality Technology & Quantitative Management*, 2006 Issue 3, pp. 347-359

ACTUALLY LCL=L and UCL=U

Another statement by INCONPETENTS

To construct a t chart, we determine the control limits based on a false alarm rate (α) of 0.0027, equaling that of an individual chart of normal data, and use the median as the centreline". Whenever historical estimates are not available, the scale parameter θ can be estimated using maximum likelihood. …. because both control limits and the centerline are functions of solely θ, the invariance property of MLEs the estimates are 0.00135 t̄, 6.60773 t̄, and log(2) t̄." .

$$LCL_T = 0.00135\,\bar{t}, \qquad UCL_T = 6.60773\,\bar{t}$$

E. Santiago, J. Smith, Control charts based on the Exponential Distribution, Quality Engineering, Vol. 25, Issue 2, 85-96

ACTUALLY LCL=L and UCL=U

Typical statement by

Suppose LCL and UCL denote the lower and upper control limits of the Phase t_r-chart respectively. Then for a given false alarm rate (FAR) α_0, they can be obtair from $P(T_r < LCL|IC) = P(T_r > UCL|IC) = \alpha_0/2$ according to the equal tail probabilit approach. Thus, we have (see also Kumar and Baranwal (2019))

$$\mathrm{LCL} = \frac{\chi^2_{2r,\alpha_0/2}}{2\lambda_0} = \frac{A_1}{\lambda_0} \text{ and } \mathrm{UCL} = \frac{\chi^2_{2r,1-\alpha_0/2}}{2\lambda_0} = \frac{A_2}{\lambda_0}$$

where $A_1 = \frac{\chi^2_{2r,\alpha_0/2}}{2}$, $A_2 = \frac{\chi^2_{2r,1-\alpha_0/2}}{2}$ are the design constants and λ_0 is the known or spe fied IC rate parameter value. The $\chi^2_{2r,a}$ denotes the a-th quantile of the chi-square dis bution with $2r$ degrees of freedom. The center line (CL) of the t_r-chart can considered as the median of the IC distribution of T_r and it is given by $\mathrm{CL} = \frac{\chi^2_{2r,0.5}}{2\lambda_0}$.

TBE data following Exponential distribution are with r=1

ACTUALLY LCL=L and UCL=U

Excerpt 2. Typical wrong formulae picked from the *"ocean full of errors…"* (<u>8 authors</u>)

Peer Reviewers and readers should practice "*metanoia*" [6] (Deming 1997) and remember his statements [5] "*Management need to grow-up their knowledge because experience alone, without theory, teaches nothing what to do to make Quality*" and "*The result is that <u>hundreds of people are learning what is wrong</u>. I make this statement on the basis of experience, seeing every day the* ***devastating effects of incompetent teaching and faulty applications.***" [Deming (1986)]

V. INDIVIDUAL CONTROL CHARTS (I-CC) AND EXPONENTIALLY DISTRIBUTED DATA. PART 2

Now we see that Reliability Integral Theory (RIT) solves the problem of computing the Control Limits for Control Charts, especially for Individual Control Charts, I-CC_TBE, for Time Between Events exponentially distributed data.

The previous section highlighted that many errors are made by PR actioners (also professors) who do not know the Theory. The readers can usefully read also [17].

We saw that <u>Minitab "T Charts" are wrong</u>; the same is for the method suggested in the paper Dovoedo Y. H. and Chakraborti S., "Boxplot-based Phase I Control Charts for Time Between Events",*Quality and*

Reliability Engineering International (*QREI*):wrong. See Fig. 13. The paper *Boxplot-based Phase I Control Charts for Time Between Events*use the same data (in Table 2). The authors (DC) write "*As an illustration, consider the example in Montgomery in which a chemical engineer wishes to control the average time between failures of a valve. She observed 20 times between failures for this valve. JC use these data, …, as an illustration of their two-sided control chart. Note that the data with all the 20 observations do not fail the Anderson–Darling test for exponential distribution. From Minitab, the Anderson–Darling statistic is found to be 0.53 with a P-value = 0.44.*" Notice that JC are the authors of another wrong paper.

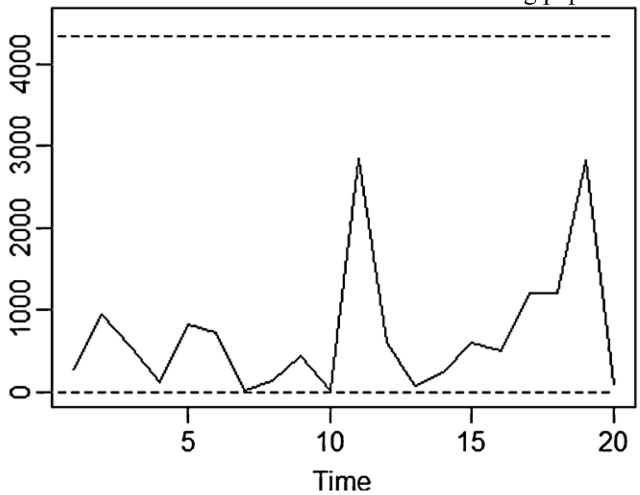


Figure 13. Chart of Montgomery data analysed by Dovoedo et al., "Boxplot-based …Control Charts for Time Between Events",*Quality and Reliability Engineering International*,2011

They obviously find the process "In Control": Fig. 13. The same happened with Minitab: figure 10. One gets the same result by using R, JMP, SAS and SPSS: a very good situation!!!

Using RIT the author could draw the Fig. 14, where the reader can see both the wrong Control Limits of the Control Chart from Minitab *and the right Lower Limit (the dotted line) forced on the graph, by FG*. The data are named Valve_TTF (Time To Failure).

The Fig. 14 is very important: it shows the wrong Control Limits [LCL, UCL] derived from the formulae (10), which are valid when the data are normally distributed, and the *right correct LCL* (the *dotted line for TBE*) *computed with RIT*. The "original" Minitab 20&21 I-Chart shows two "wrong" Out Of Control points (above the wrong UCL) that does not actually exist.

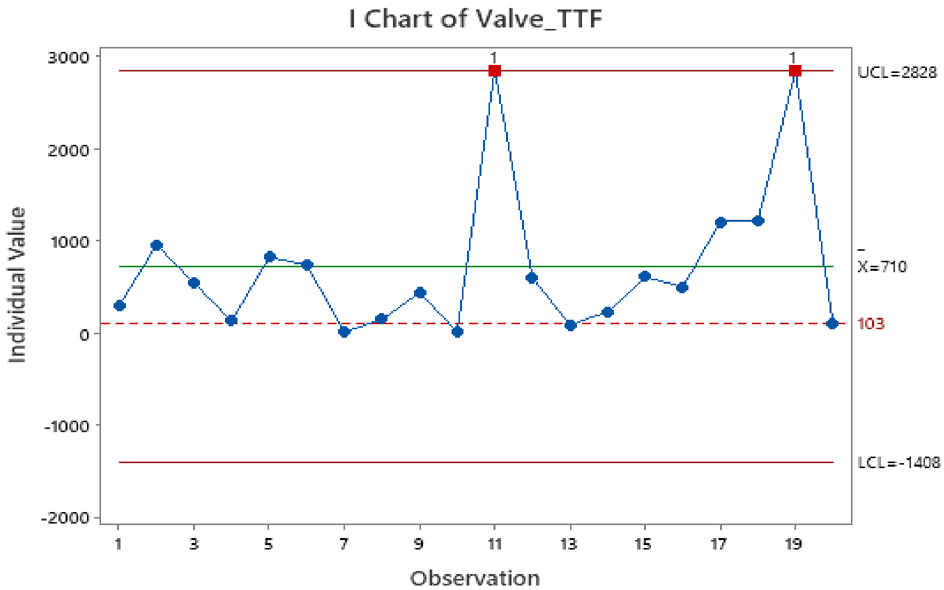


Figure 14. (F. Galetto) I-Chart (Control charts) for valves data [Montgomery book]. The new release Minitab 20&21 used. *The dotted line is the right correct LCL when RIT is used*

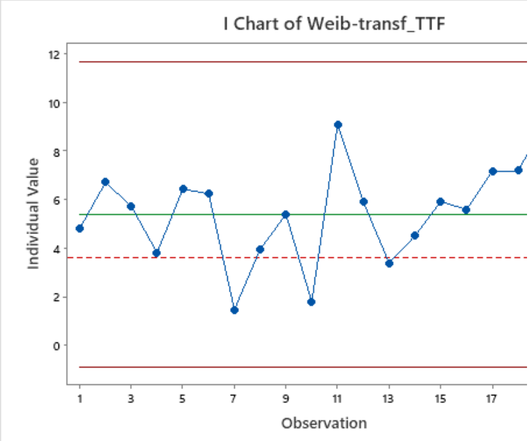
This is shown also in the Fig. 1: being the data exponentially distributed, also the ranges are exponentially distributed (Galetto books).

Moreover it does not show the real Out Of Control points below the dotted line: it shows them because the author forced the software to draw the *correct LCL (the dotted line)*. The Minitab LCL is wrong, as well.

Therefore, we see that actually the process is "Out Of Control" (as said in the Introduction).

All the wrong methods in the ***"****ocean full of errors by ….****"*** (see the Excerpt 2, with 8 authors) cannot find that actually the process is "Out Of Control".

We can transform the Exponential data into Weibull data, as suggested by D. C. Montgomery, who used the idea of Nelson (Fig. 14b), and by the Johnson's transformation (Fig. 14c).



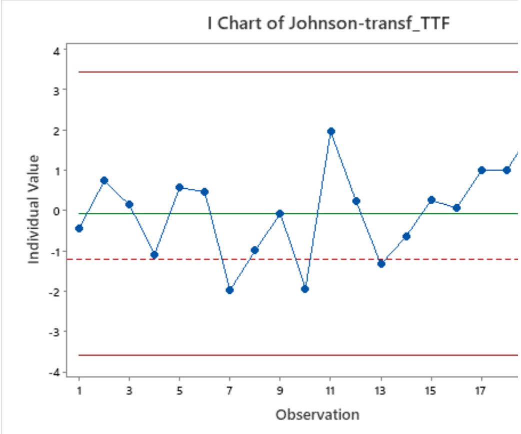


Figure 14b. Figure 14c.

Figures 14b and 14c. (F. Galetto) I-Chart (Control charts) for valves data [transformed into Normal data, by Weibull transformation and by the Johnson's transformation]. Releases Minitab 20&21 used. *The dotted line is the right correct LCL when RIT is used*

We saw that all the Minitab T-Charts suggests us that the process is In Control.

The dotted lines show, on the contrary, that the process is Out Of Control.

The related I-Chartsare in Fig. 14b and 14b: they show the wrong Control Limits [LCL, UCL] derived from the formulae (10), now valid because the transformed data are normally distributed, and the *right correct LCL* (the *dotted line*) *computed with RIT*.

T Minitab 20&21 I-Charts do not show the real Out Of Control points (transformed by Weibull and Johnson's transformations!) below the dotted line: it shows them because the author forced the software to show the dotted line.

Therefore we clearly see that we need the ***right and scientific method*** to analyse the data and derive the correct Control Charts: data transformations can hide the truth.

If the reader considers that the author asked many [>>100] ‴Statisticians and Certified Master Black Belts and Minitab users (you can find them in various forums such as ReasearchGate, iSixSigma, Academia.edu, Quality Digest, … and in several Universities)‴ and *nobody* could solve scientifically the cases, he has the dimension of the problem.

The author wrote also to the editors of *Quality and Reliability Engineering International*, *Quality Technology & Quantitative Management*, *Journal of Quality Technology*, *Reliability Engineering & System Safety*, *Quality Engineering*, *Journal of Statistics and Management Systems*, *Int. Journal for Research in Applied Science & Engineering Technology* and to the management of Minitab Inc. about their wrong T-Charts: the letters are not yet been published: the papers are wrong and, obviously, neither the Editors nor the Minitabmanagement cannot acknowledge that: this is another point of risk for the author!

Nobody took care of the problem and nobody provided the correct way to compute the Control Limits (LCL and UCL) of the Control Charts for the TBE data (see the "*ocean…*").

This disaster is due to a very diffused error about the concept of Confidence Intervals: the Control Limits (LCL and UCL) of any type of Control Charts are actually the limits of the Confidence Intervals, with Confidence Level (CL) 0.9973 (i.e. 0.0027 risk of "possible" wrong decision).

All of them make confusion between the concepts of L and U with the LCL and UCL….This proves the truth of Deming's statements [1996] "*The result is that hundreds of people are learning what is wrong.*","*It is a* *hazard to copy", "It is necessary tounderstand the theory of what one wishes to do or to make."*, *"**There is no substitute for knowledge**"*.

To find the correct limits the reader should apply RIT, Reliability Integral Theory [18-26] (Galetto 1981-94, 1982-94, 2010, 2015, 2016).

RIT was devised in 1977 by Fausto Galetto when he was working at FIAR (a division of General Electric). After that he was Reliability Manager at Fiat Auto [now Stellantis] and, at the same time he was Professor of Reliability Methods at Padua University (Faculty of Statistics) [18-19] (Galetto 1981-94, 1982-94).Later he was Quality and Reliability Director at IVECO and then Professor of Quality Management at Turin Politecnico.

Let's consider a "stand-by" system, made of n "identical" units, whose reliability is (for each unit)

$$R(t;\mu,\sigma) = \int_t^\infty f(s;\mu,\sigma)ds = P[T \geq t;\mu,\sigma] \qquad (12)$$

where $f(t;\mu,\sigma)$ is the probability density function (pdf) of failure of the Random Variable T, the Time To Failure (TTF); μ and σ are two parameters of the pdf.

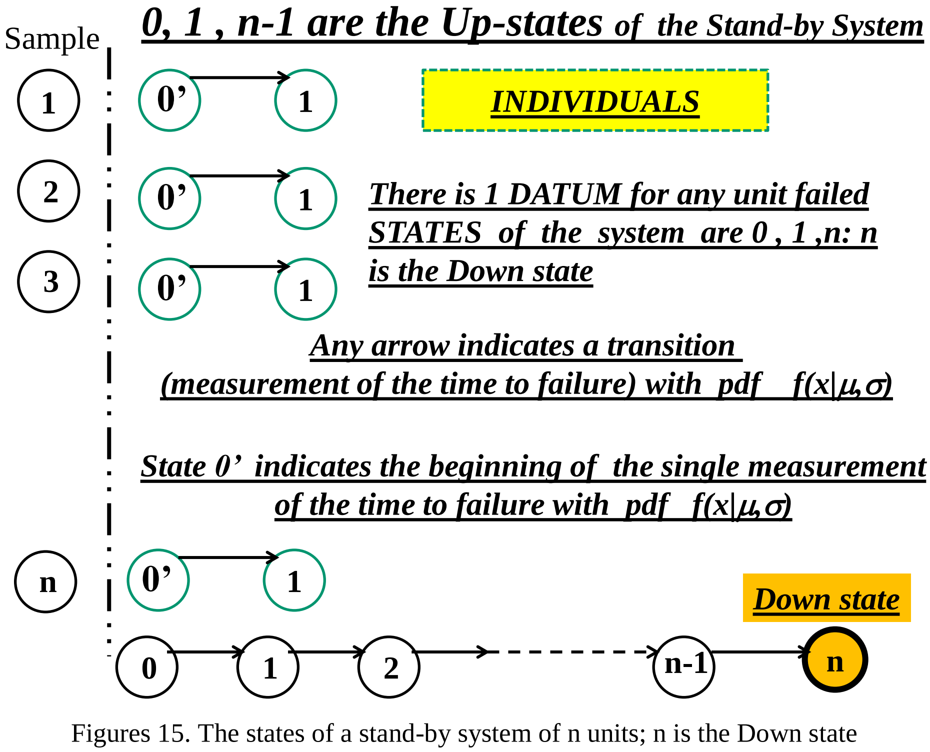
Look at Fig. 15. A number n of units makes the system: the units work one at a time.

At each failure of a unit, say j, a subsequent new unit j+1 starts operating.

We define the "interval reliability", for the interval r------t, as the probability that the unit does not fail before time t, given that it did not fail before time r (we do not show the parameters in the last formula)

$$R(t|r;\mu,\sigma) = P[T \geq t|T \geq r;\mu,\sigma] = \bar{W}(t|r) \qquad (13)$$

We define the "instantaneous transition probability" from the state j to the state j+1, $b_{j,j+1}(s|t_j)ds$, as the probability that a failure happens in the interval s------s+ds, given that the j-the unit failed at time t_j.



Figures 15. The states of a stand-by system of n units; n is the Down state

The "stand-by" system is reliable (performs its intended function) until there is a new unit that can start operating; the system is failed (Down state) when the last unit fails. We name $R_0(t|t_0)$ the reliability of the system when it starts, in the state 0, with the 1st unit at time t_0; we indicate with $R_0(t|t_0)$ the reliability associated to the state 0; if there are no failures in the interval t_0------t, then we have $R_0(t|t_0) = \bar{W}_0(t|t_0)$. If, on the contrary, a failure happens (transition from 0 to 1) at time s<t the system goes to the next state with probability $b_{0,1}(s|t_0)ds$; then the 2nd units starts and to have the system "well working" at time t, we must have the new unit with reliability $R_1(t|s)$ for the remaining interval s------t. The reliability of the system is now given by $R_0(t|t_0) = \bar{W}_0(t|t_0) + \int_{t_0}^t b_{0,1}(s|t_0)R_1(t|s)ds$ .We can repeat the same argument for the states 1,…, n-1 (see the Fig. 15); so we have the set of Integral Equation

$$R_j(t|t_j) = \bar{W}_j(t|t_j) + \int_{t_j}^t b_{j,j+1}(s|t_j)R_{j+1}(t|s)ds,$$

$$\text{for } j = 0, 1, \dots, n-1, \text{ and } R_{n-1}(t|t_{n-1}) = \bar{W}_{n-1}(t|t_{n-1}) \quad (14)$$

Let $t_0=0$: the system we consider starts in State 0, at time 0: all the units are reliable.

The general formula, in matrix form, of the reliability of a n units stand-by system is given [18-26] (Galetto 1981-94, 1982-94, 2010, 2015, 2016)by (15)

$$R(t) = \bar{W}(t) + \int_0^t B(s)R(t-s)ds \quad (15)$$

where R(t) is the vector of the reliabilities $R_i(t)$ associated to the Up-states, i=0, 1, …, n-1, B(s) is the square matrix of the kernels $b_{ij}(s)$ [related to the transition probabilities $b_{ij}(s)ds$] between the Up-states and $\bar{W}(t)$ the diagonal matrix of the probabilities of remaining in each Up-state, for the time mission t.

If the transitions kernels $b_{j,j+1}(s|t_j) = \lambda_j \exp(s - t_j)$ are exponential functions, being A the matrix of the *constant transitions rates* we get, from (15), the EQUIVALENT matrix equation [u is the column vector with all entries 1]

$$R(t) = u + \int_0^t AR(t-s)ds \quad (16)$$

When we consider the exponential kernels, (15) [and (16)] are the *fundamental system of the Reliability Integral Theory, for Markov processes*.

In F. Galetto's books [18-26] (Galetto 1981-94, 1982-94, 2010, 2015, 2016) RIT is extended to Reliability Tests, for estimating parameters and testing of hypothesis, for any type of transition kernels. This solves the *Time Between Events* Control Charts Problem. We use the concept of «*system associated to a reliability test*»: it is a system of (n units) n-1 Up-states with transition rate λ, where λ=1/θ and θ is the MTTF of any units; we want to estimate θ. We can look at the evolution (with the elapsed time t) of the test as the evolution of a standby system for the interval 0---t: see the figure 14. The reliability of any item [n are the items on test] determines the instants of the transitions of the "*system associated (to the reliability test*"; the state n is the "down-state (orange in the figure 14)", at which the test is over. Therefore, for the Reliability Tests, we get the following fundamental system of Integral Theory of Reliability Tests[F. Galetto, *holding for any distribution of the time to failures* of the units], for i=0, 1, …, n-1, where we assume that r is the entrance time instant when we begin observing the system

$$R_i(t|r) = \bar{W}_i(t|r) + \int_r^t b_{i,i+1}(s|r)R_{i+1}(t|s)ds \quad (17)$$

In matrix form [R(t|r) is the vector of the reliabilities, B(s|r) is the matrix of the kernels]

$$R(t|r) = \bar{W}(t|r) + \int_r^t B(s|r)R(t|s)ds \quad (18)$$

Important is the first component $R_0(t)$ of the vector R(t|0), which is the probability that the physical sample does not experience the n^{th} failure during the interval 0-----t (t is the duration of the reliability test started at r=0). At the end of the reliability test, at time t, we know the instants of the failures t_i of the empirical sampleD={t_1, t_2, …, t_{n-1}, t} and we can make explicit the equations as follows [t_0=0, the start of the test]:

$$R_j(t|t_j) = \bar{W}_j(t|t_j) + \int_{t_j}^t b_{j,j+1}(s|t_j)R_{j+1}(t|s)ds,$$

$$\text{for } j = 0, 1, \dots, n-1, \text{ and } R_{n-1}(t|t_{n-1}) = \bar{W}_{n-1}(t|t_{n-1}) \quad (19)$$

From the equations (18) we compute the determinant of the integral system **detB(s|r)** [that depends on the parameter λ]. In the case of a reliability test for estimating the failure rate λ of the identical units, we have

$$\det[B(s|r;\ \lambda, D) = \lambda^{n-1}\exp[-tot(t)] \quad (20)$$

where $tot(t) = \sum_1^{n-1} t_i + (t - t_{n-1})$ is the "Total Time on Test" generated by n items tested until the n-1 failure with the test ending at time t. At the end of the reliability test we have the *empirical sample* D; the integral equations are constrained by D; if we want that our "total sample of n items" has the maximum probability, given the constraintD, we get the equation $\frac{\partial lndetB\ (s|r;\lambda,D)}{\partial\lambda} = {(n-1)}/{\lambda} - tot(t) = 0$.

This is exactly the same equation we can get with the Maximum Likelihood method.

Let's consider a sample of "n items" and let all the units fail: the reliability test is censored at the number of failures n. We *estimate* the mean time to failure θ=1/λ, of each unit, by the ratio «*total time on test*$tot(t) = \sum_1^n t_i$»/«*number of failures, n*». The estimator of θ in *censored* reliability tests (censored at the number of failures n) is the Random Variable $\widetilde{\Theta} = T(t)/n$, ratio of the RV "total time on test $T(t) = \sum_1^n T_i$ and the number of failures n, fixed in advance" (efficient estimator); T_i is the RV "Time To Failure" of the i-th unit. The Associated System to the Reliability Test has n+1 states: 0, 1, …, n-1 are the Up-states, while n is the Down-state (see Fig. 15).

The Confidence Interval (symmetric) for the parameter θ [which is the MTTF of any unit] can be obtained by solving the two equations, with θ unknown and $\boldsymbol{t_o}$ the "known" observed$\boldsymbol{t_O} = tot(t) = \sum_1^n t_i$ determination of the RV Total Time on Test $T(t)$, whose solution is θ_L and θ_U, with Confidence Level CL=1-α

$$R_0(t_O;\ \theta_L) = \alpha/2\,, \qquad R_0(t_O;\ \theta_U) = 1 - \alpha/2 \quad (21)$$

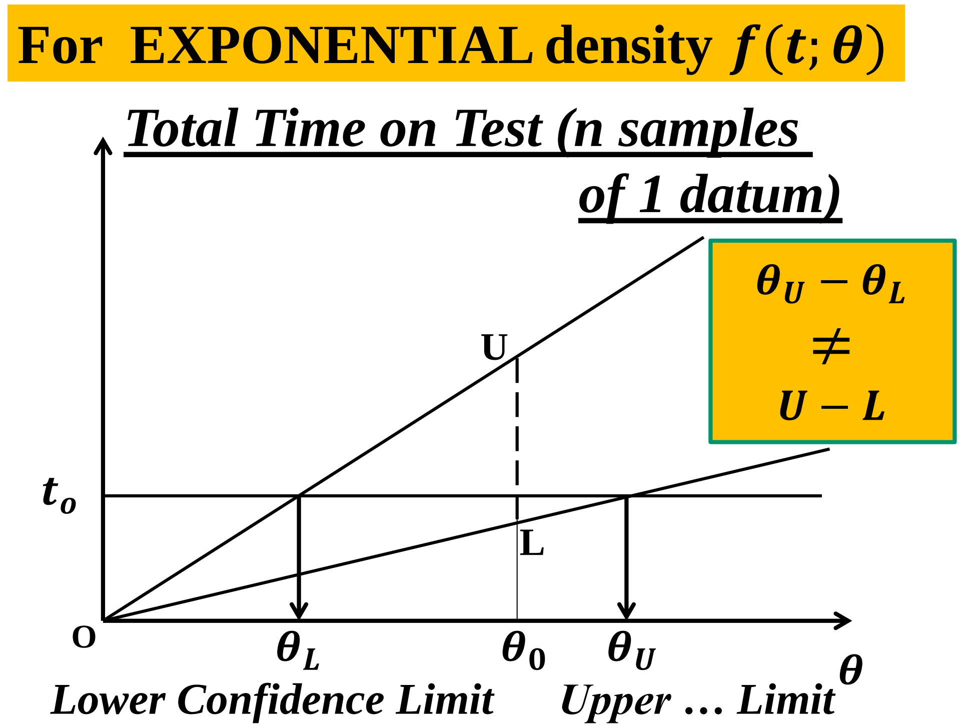


Figure 16. (F. Galetto) Confidence Interval of the MTTF. Remember that in this case k=1 (sample size)

Since the reliability of any unit is exponential $R(t|\theta)=\exp(-t/\theta)$, (see the Fig. 16) the function $t/\theta=K$ is a straight line $t=K\theta$, with angular coefficient K in the plane with abscissa θ and ordinate t: the coefficient K_1^n is related to $\alpha/2$ and n (the number of data) while K_2^n is related to $1-\alpha/2$ and n. Thus, we have two lines, passing through the origin O [with linear scale, in Fig. 16]; putting $\theta=\theta_0$ the two lines intercept the **vertical segment L----U (probability interval)**, which has probability $\pi=1-\alpha$ that the "time to failure", Random Variable T, of any unit [vertical axis named "Total Time on Test", because we consider all the data] is in the interval **L----U** when $\theta=\theta_0$. The angle $L\hat{O}U$ depends on the values $\alpha/2$, $1-\alpha/2$ and n (the number of data).

Acting as we did for the normal case (see Fig. 3), with the known quantity $\boldsymbol{t_o}=tot(t)=\sum_1^n t_i$ [observed determination of the RV Total Time on Test $T(t)$], we can draw the horizontal line intersecting the two lines through the origin: the abscissas of intersections are the two numbers θ_L and θ_U, depending on the values $\alpha/2$, $1-\alpha/2$ and n (the number of data).

The two numbers θ_L and θ_U are the Lower limit and the Upper limit of the Confidence Interval of the MTTF of each unit, with CL=$1-\alpha$.

It is evident, for any intelligent person, that the two segments **L----U** (vertical) and **θ_L----θ_U** (horizontal) are two *different* intervals with clear *different*meaning and obvious *different lengths*$\theta_U-\theta_L\neq$U-L! All the documents, known to the author, make this BIG ERROR: they confound the vertical segment **L----U**, which is a "**Probability segment**" with the horizontal segment **θ_L----θ_U**, which is a "***Confidence segment***"! See the "*ocean full of ....*" and Excerpt 2 (8 authors).

Now we see how RIT solves the Montgomery case (TBE Control Charts).

We have now to look at the Fig. 17 (similar to Fig. 16, but with different lines).

Since the reliability of any unit is exponential $R(t|\theta)=\exp(-t/\theta)$, (see the Fig. 17) the function $t/\theta=K$ is a straight line $t=K\theta$, with angular coefficient K in the plane with abscissa θ and ordinate t: the coefficient K_1^1 is related to $\alpha/2$ and 1 (the single datum) while the coefficient K_2^1 is related to $1-\alpha/2$ and 1. As before we have two lines, passing through the origin O [with linear scale, in figure 16]; putting $\theta=\theta_0$ the two lines intercept the **vertical segment L----U (probability interval)**, that has probability $\pi=1-\alpha$ that the "time to failure", Random Variable T, of any unit [vertical axis named "Time on Test", because we consider the single data] is in the interval **L----U** when $\theta=\theta_0$. The angle $L\hat{O}U$ depends on the values $\alpha/2$, $1-\alpha/2$ and 1 (the single data).

Does the number of Citations of the authors floating in the "*ocean full of errors by incompetents*" mean Quality of those authors? Is a "Quality author" an "associated editor of… (in the *ocean*)"?

Acting as we did for figure 15, with the known quantity "*mean observed time to failure*" $\bar{t_o}$ =$\boldsymbol{t}_o$/n=$tot(t)/n = \sum_1^n t_i/n$ [observed determination of the RV Mean Time on Test $T(t)/n$], we can draw the horizontal line intersecting the two lines through the origin: the abscissas of intersections are the two numbers LCL and UCL, depending on the two chosen values α/2, 1-α/2 and 1 (the single data).

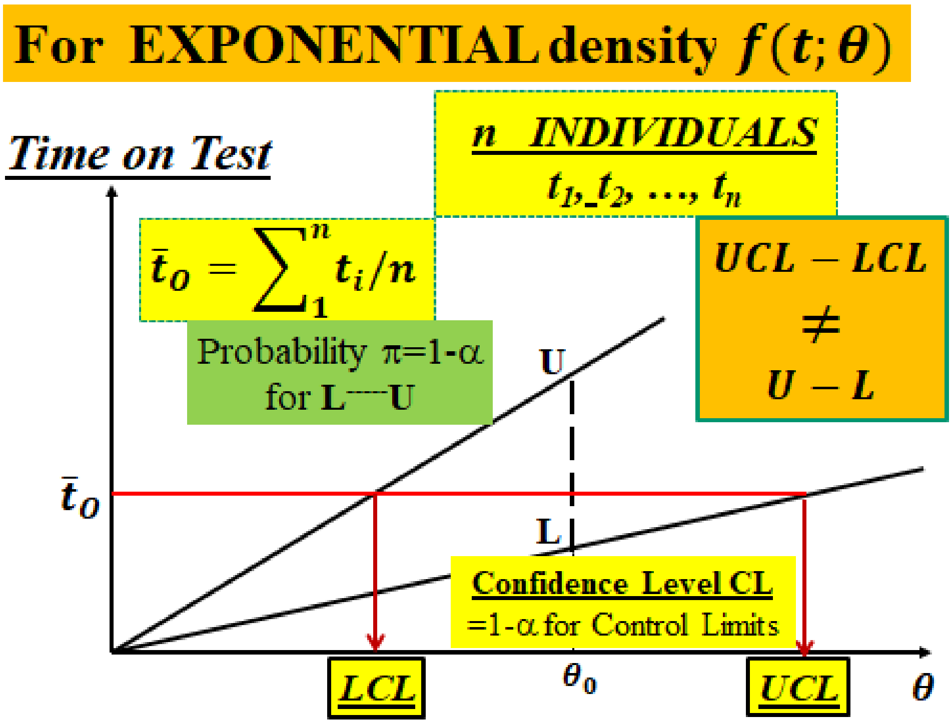


Figure 17. (F. Galetto) LCL and UCL for the TBE Control Chart (exponentially distributed data), using RIT. Remember that in this case k=1 (sample size)

As a matter of fact, in the I-CC, the Control Limits LCL and UCL must be consistent with the "individual" times to failures: we want to analyse if they are significantly different from the "true mean θ", estimated by the "*mean observed time to failure*" $\bar{t_o}$ =$\boldsymbol{t}_o$/n. Therefore the Control limits are the values satisfying the two equations (21) with $\boldsymbol{t}_o$ replaced by $\bar{t_o}$ =$\boldsymbol{t}_o$/n, that is the two equations (22) for any single unit; so we have 20 Confidence Intervals [all equal, by solving formulae (22)], given $\bar{t_o}$ and CL=1-α [CL=0.9973],

$$R(\bar{t_O};\ LCL) = \alpha/2\,,\qquad R(\bar{t_O};\ UCL) = 1-\alpha/2 \tag{22}$$

Remember that in this case k=1 (sample size): I-CC!

With the Montgomery's data the picture of Fig. 17 would be unreadable. Hence we transform logarithmically both the axes and get the Fig. 18. LCL and UCL are the abscissas of points intercepted by the horizontal line$\bar{t_o}$ =$\boldsymbol{t}_o$/n with the two lines passing through - ∞ (the transformed origin of figure 16): hence, ***Confidence interval***.

The n=20 lifetimes in table 2 can be considered as the "transition times" (failure times, exponentially distributed) between the states of a stand-by system of 20 units: the Up-states are 0, 1, …, 19, and 20 (n) is the Down-state; t_i is the "time to failure" from state i-1 to state i: they are the "individuals". The reliability R_0(t|θ) [the system reliability R_0(t|θ) given the parameter θ)] is, as well, the Operating Characteristic Curve (Galetto 1981-94, 1982-94, 2010, 2015, 2016) of the reliability test, given t: the pdf of any transition is $f(t;\ \mu,\sigma) = (1/\theta)\exp(-t/\theta)$; [see Fig. 15].

It is evident, for any intelligent person, that the two segments **L----U** (vertical) and ***LCL----UCL*** (horizontal) are two *different* intervals with clear *different*meaning and obvious *different lengths* UCL-LCL≠U-L! All the documents, known to the author, make this BIG ERROR: they confound the vertical segment, which is a ″**Probability segment**″ with the horizontal segment, which is a ″***Confidence segment***″!

Does the number of Citations of the authors floating in the "*ocean full of errors…*" mean Quality of those authors? Greater number than greater Quality? NO, see figure 19.

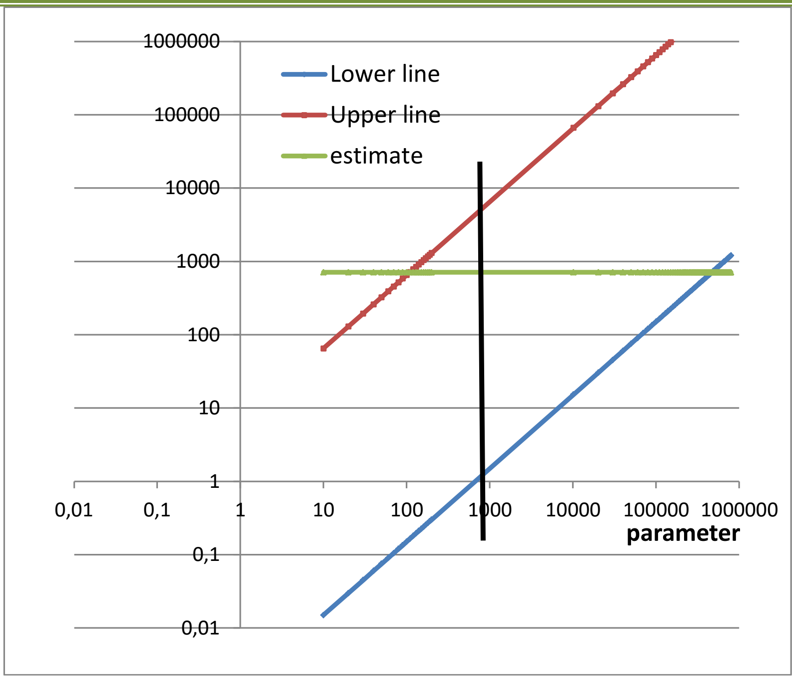


Figure 18. (F. Galetto) LCL and UCL for the TBE Control Chart of Montgomery data, using RIT. [logarithmic scales] <u>Remember that in this case k=1 (sample size)</u>

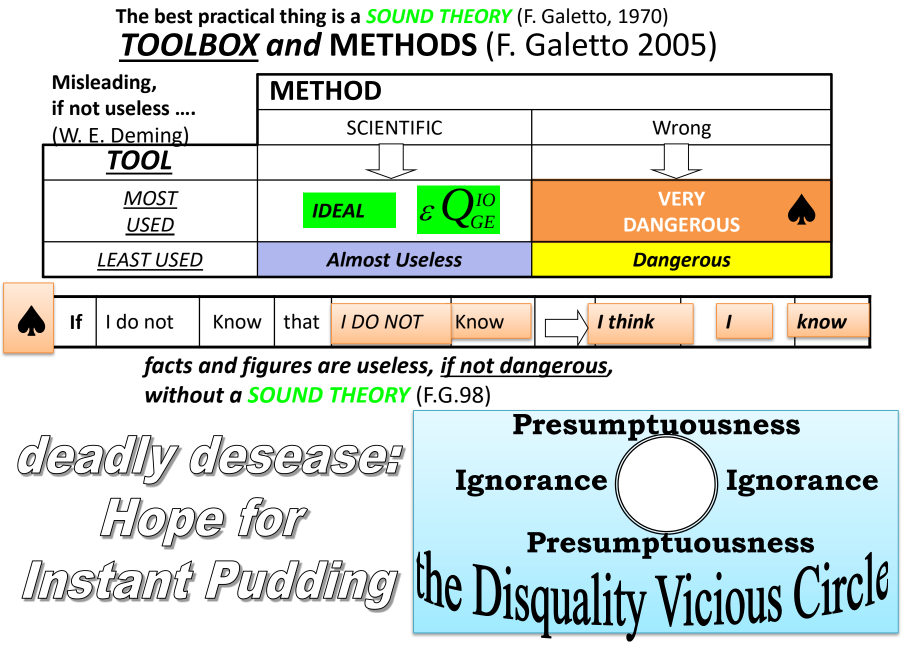


Figure 19. Knowledge versus Ignorance, in Tools and Methods, with the Dis-quality Vicious Circle

The reader perhaps thinks that the problem of wrong decisions could happen only for the Montgomery case.

Actually all the papers in the "*<u>ocean full…</u>*" have the same problem: wrong Control Limits of the Control Charts, both for the mean and for the variance.

Their authors should have avoided running in the "Dis-quality Vicious Circle" and they should have taken into account the problem of "Knowledge versus Ignorance, in Tools and Methods" (Fig. 19). See in particular the papers in *<u>QREI</u>*!

If those authors had considered the concepts in the figure 18, many readers could learn good ideas instead of finding the wrong cases in the next session.

They should have had the same attitude of J. Juran who, at the Vienna Conference, mentioned the paper "*Quality of methods for quality is important*" [27] (Galetto 1989) in the plenary session.

The ideas in this section are very useful and can be used also for the Control Charts of the variance.

VI. CASES FROM "PEER REVIEWED" PAPERS

Now we see how RIT can solve a case, found in the paper "Improved Phase I Control Charts for Monitoring Times Between Events" published by *Quality and Reliability Engineering International* (2014 and found online, 2021, March).

The two authors provide a wrong solution found neither by the Peer Reviewers nor by the Editor. Nevertheless, they write in their Acknowledgements*: The first author's post-doctoral fellowship is supported by the South African Research Chairs Initiative (SARChI) award to the second author at the University of Pretoria, in South Africa. Partial support is also provided by the Department of Statistics, University of Pretoria. The authors would like to thank Dr. Douglas Montgomery, Co-editor, for his interest and encouragement.*

We begin with the authors' Abstract, where they say:

In many situations, the times between certain events are observed and monitored instead of the number of events particularly when the events occur rarely. In this case, it is common to assume that the times between events follow an exponential distribution. Control charts are one of the main tools of statistical process control and monitoring. Control charts are used in phase I to assist operating personnel in bringing the process into a state of statistical control. In this paper, phase I control charts are considered for the observations from an exponential distribution with an and out-of-control performance of the proposed chart. It is seen that ***the proposed charts are considerably more in-control robust than*** *two competing charts and have comparable out-of control properties. Copyright © 2014 John Wiley & Sons, Ltd.*The data, given there, are:

Table 3. Time between failures data ("Improved Phase… for Monitoring TBE".).

1.24	6.69	9.77	1.23	14.03	18.07	3.90	13.61	18.47	12.85
52.32	14.75	4.69	0.18	13.61	4.57	0.28	7.08	12.00	5.15
6.09	20.41	5.93	19.03	13.65	6.37	2.06	3.30	6.91	12.08

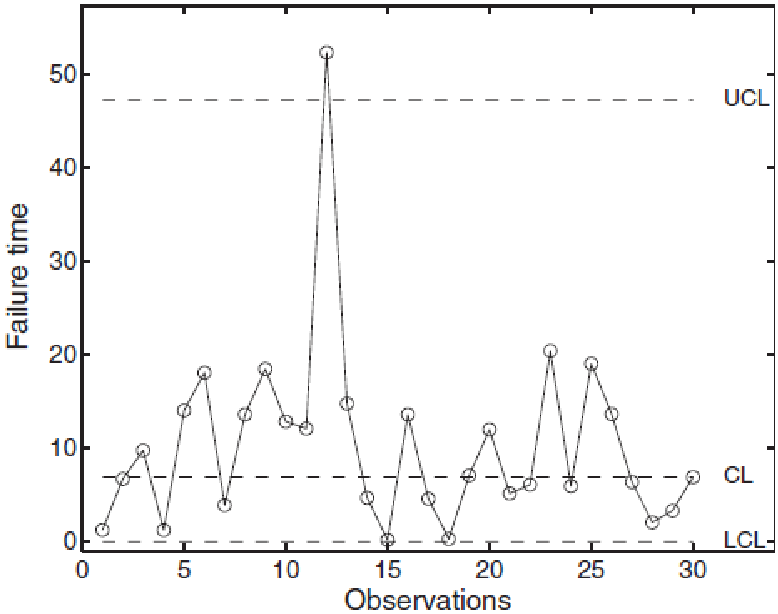


Figure 20. [Excerpt] Control Chart from "Improved Phase… for Monitoring TBE". *Remember that in this case k=1 (sample size)*

The authors say: "*Table … shows a set of 30 failure time data generated from a Poisson distribution with a mean of 0.1. For these data, n = 30, l=8, m=15, and u=23. We monitor these data with the proposed phase I chart. The center line for the proposed two-sided control chart is CL=X(15)=6.91, and the lower and UCL are given by LCL=-53.9213 and UCL=47.2320. Because LCL<0, we set the LCL as LCL=0. It can be seen from Figure* (our 19) *that* the eleventh observation 52.32 plots outside the UCL*, which indicates an OOC situation that needs further investigation. Note that for these data, neither the Dovoedo and Chakraborti, nor the Jones and Champ control chart indicates any OOC situation.*" Their Control Chart is in figure 19. Notice that the "wrong" Control chart shows an Out Of Control (OOC) situation that should not be there and various In Control (IC) that should not be there…

Now we use RIT.

As done in the previous section, the n=30 TBE can be considered as the "transition times" (failure times, exponentially distributed) between states of a stand-by system of 30 units: the Up-states are 0, 1, …, 29, and 30 is the Down-state; t_i is the "time to failure " from state i-1 to state i. $R_0(t|\theta)$ is the system reliability for the interval 0¯¯¯t, given θ, and it is, as well, the Operating Characteristic Curve of the reliability test, given t.

At the end of the test we know t_o the observed Total Time on Test. The Control Limits LCL and UCL must be consistent with the "individual" TBE: we want to analyse if they are significantly different from the "mean observed time to failure" $\bar{t_o}=t_o/n$. Therefore the Control limits are the values satisfying the two equations (21) with t_o replaced by $\bar{t_o}=t_o/n$, that is two equations (22) for any single unit; so we have 30 Confidence Intervals [all equal, by solving the right formulae], given $\bar{t_o}$ and CL=1-α, $R(\bar{t_0}; LCL) = \alpha/2 \quad and \quad R(\bar{t_0}; UCL) = 1-\alpha/2$.

Comparing both the Fig.20 and 21, it becomes very clear that the Control Chart from "*Improved Phase… for Monitoring TBE*" presents *5 errors* about OOC.

Reader, could you think that the paper "Improved Phase… for Monitoring TBE" is scientific and this one is not?How can the Control Chart from "Improved Phase… for Monitoring TBE" be good?

See their "absurd" Concluding remarks.

In this paper, phase I control charts are considered for observations from an exponential distribution with an unknown mean. The proposed charts are based on the median and hence hold the IC robustness property well. In fact, the proposed charts are shown to be more IC robust than both the Jones and Champ and the Dovoedo and Chakraborti charts currently available in the literature. Further work is necessary on the OOC performance of these charts.

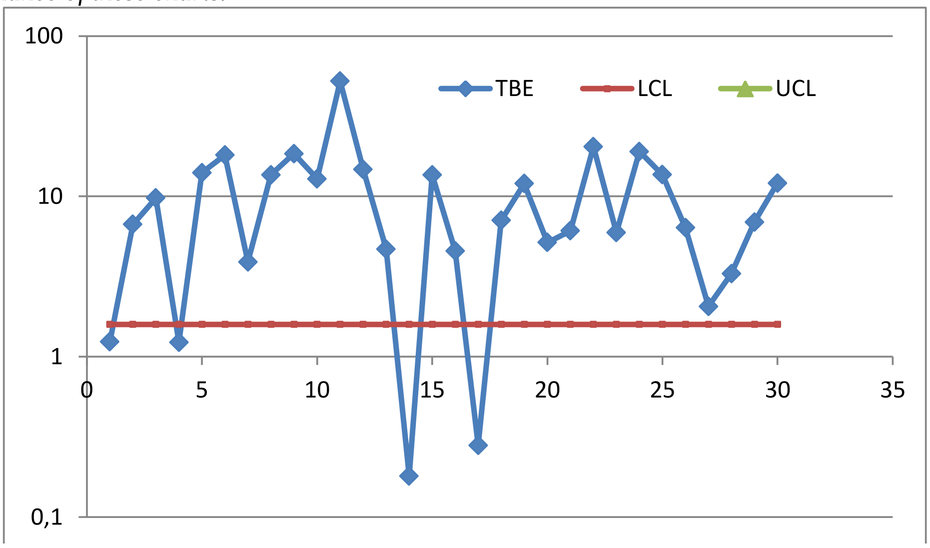


Figure 21. Control Chart of the data from "Improved Phase… for Monitoring TBE"; vertical axis logarithmic; UCL is >100. RIT used (F. Galetto). Remember: k=1 (sample size)

Simulations (five millions!) show that < 5% of the computations are correct…

We agree with those authors that "*Further work is necessary on the OOC performance of these charts*": the further Work must be to STUDY (see Deming!) to avoid "Huge costs of DIS-quality applications/decisions"… See FAUSTA VIA, SPQR principle and CARE principle, in Conclusion.

We ask the reader: do you think that these finding are not supported by Theory and Methods?

Consider now the paper "Control charts based on the Exponential Distribution", published in *Quality Engineering*. Both the authors have good qualifications: E. Santiago is a *technical training specialist* (B.S., M.Eng. and Ph.D.) and J. Smith is a *statistician*, (certified Lean Six Sigma Black Belt. and Chair-Elect of the ASQ Statistics Division); they both were (are now?) working at Minitab, Inc. Both the methods seen before, the (Minitab) T Charts and the Box-plot, *compute WRONG Control Limits!*

Therefore the process is considered In Control, ***but it is not***: see Fig. 22. The data are in Table 4.

The data analysis with RIT provides quite a different picture: the Process Out of Control ….

Again a Peer Reviewed paper in a "good Journal" …

Table 4 Urinary Tract Infection Data

0.57014	0.03333	0.22222	0.03819	0.25000	0.08681	0.12014	0.32639	1.08889
0.07431	0.08681	0.29514	0.24653	0.40069	0.40347	0.11458	0.64931	0.05208
0.15278	0.33681	0.53472	0.29514	0.02500	0.12639	0.00347	0.14931	0.02778
0.14583	0.01389	0.15139	0.11944	0.27083	0.18403	0.12014	0.24653	0.03472
0.13889	0.03819	0.52569	0.05208	0.04514	0.70833	0.04861	0.04514	0.23611
0.14931	0.46806	0.07986	0.12500	0.13542	0.15625	0.02778	0.01736	0.35972

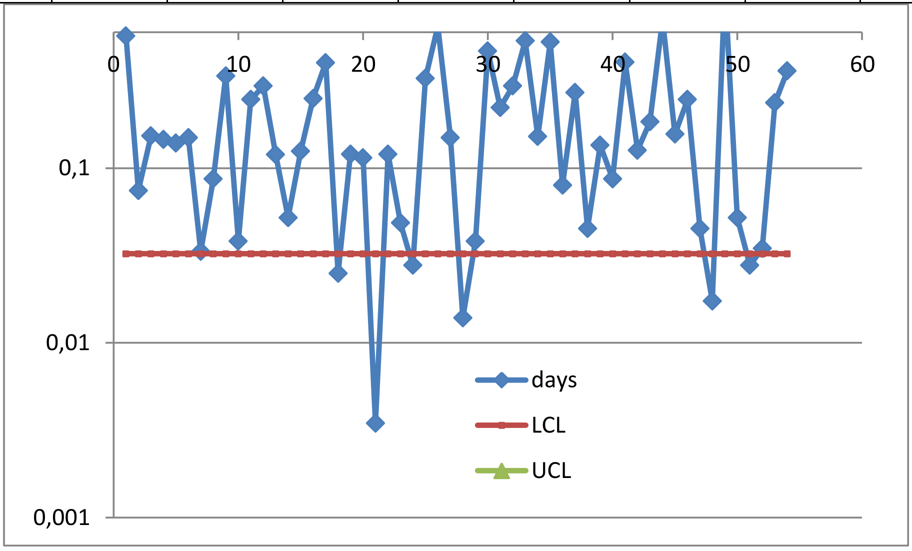


Figure 22. Control Chart of Minitab authors' paper data (Urinary); vertical axe logarithmic. ***RIT used*** (F. Galetto). *Remember that in this case k=1 (sample size)*

Consider also the paper "Some effective control chart procedures for reliability monitoring" (2002), published in *Reliability Engineering & System Safety*.Qualified authors Xie, M., Goh, T. N., Ranjan, P., paper Peer Reviewed by qualified Referees. Again WRONG Control Limits!See Fig. 23.

It is clear wrong methods induce their users to take wrong decisions, in spite of the authors' qualifications. The data of "Time between failures of a component" [TBF(h)], are given in table 5.

At least 10% of the data are Out Of Control: Xie et al. did not found that. A very good result for a "scientific" paper Peer Reviewed and published in the Good Journal *Reliability Engineering & System Safety*.

Table 5 Time between failures of a component (from Xie, Goh, Ranjan ["*ocean…*"])

Failure	TBF	Failure	TBF	Failure	TBF	Failure	TBF	Failure	TBF
1	30.02	7	5.15	13	3.39	19	1.92	25	81.07
2	1.44	8	3.83	14	9.11	20	4.13	26	2.27
3	22.47	9	21,00	15	2.18	21	70.47	27	15.63
4	1.36	10	12.97	16	15.53	22	17.07	28	120.78
5	3.43	11	0.47	17	25.72	23	3.99	29	30.81
6	13.2	12	6.23	18	2.79	24	176.06	30	34.19

A very good result for a Peer Reviewed paper! The two Peer Reviewers (of the paper) did not know the Theory. *"It is necessary to understand the theory of what one wishes to do or to make."* [6] (Deming 1997)

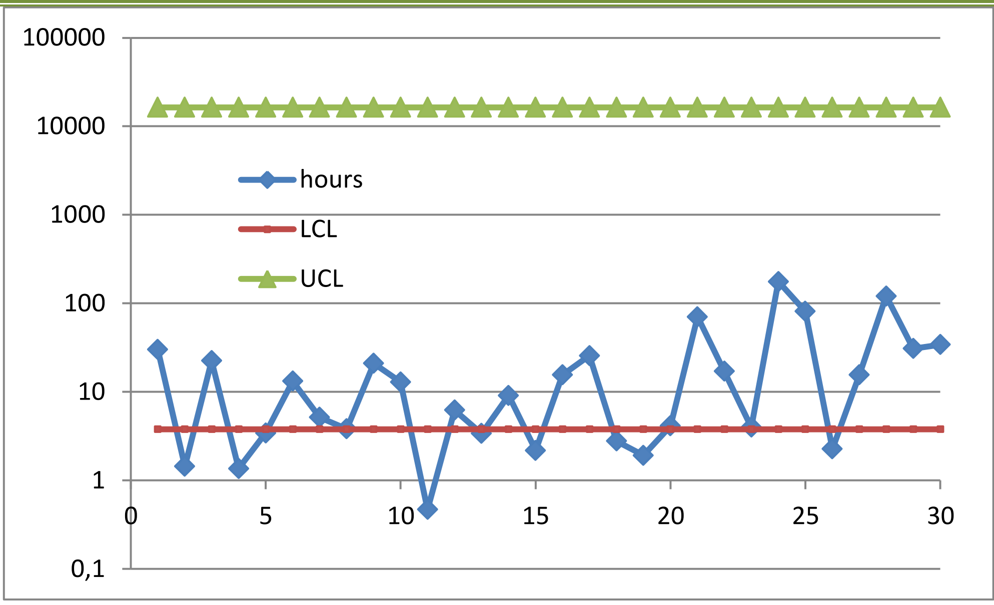


Figure 23. Control Chart of Xie et al. TBF data; vertical axe logarithmic. ***RIT used*** (F. Galetto). *Remember that in this case k=1 (sample size)*

In the paper N. Kumar, A. C. Rakitzis, S. Chakraborti, T. Singh (2022), "*Statistical design of ATS-unbiased charts with runs rules for monitoring exponential time between events*", we find a new wrong case copied from Santiago and Smith (2013).

Notice that the Control Limits are wrong.

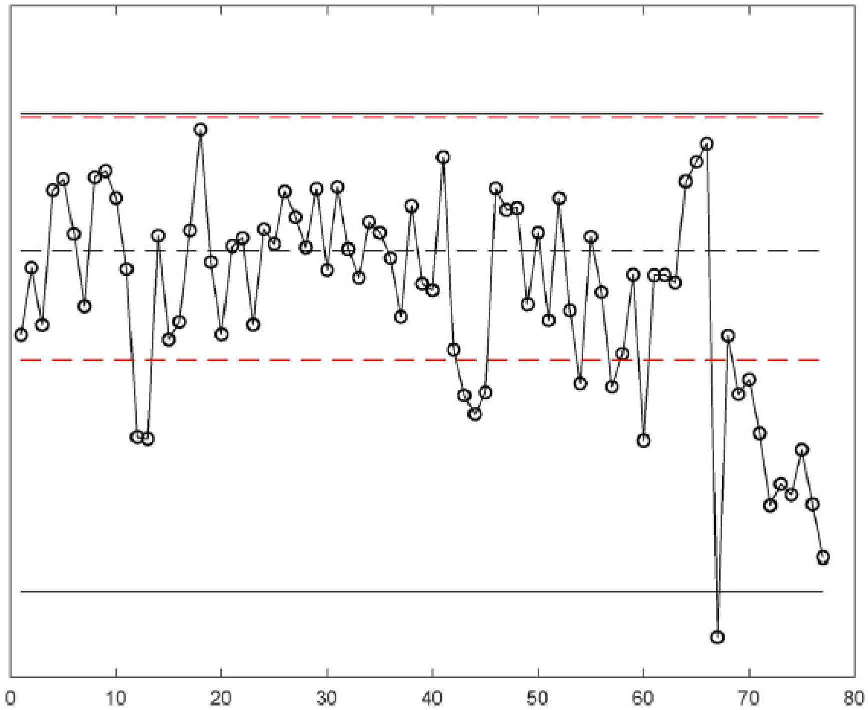


Figure 1. Control limits of ATS-unbiased t_1-charts without a runs rule scheme (black and solid lines) and with the $\{1/1, M : 3/4\}$ scheme (red and dashed lines).

Figure 24. Control Chart of Kumar, Rakitzis, Chakraborti, Singh (2022), *k=1 (sample size)*

The authors write: An example*As an illustration of the application of the proposed ATS-unbiased chart with runs rules, we consider the data provided in Santiago and Smith (2013). The data consists of 77 observations (time intervals) of registered earthquakes of magnitude of 1.0 or higher from January 24, 1978 to*

*March 25, 1980 in the general vicinity of Mount St. Helens. Santiago and Smith (2013) showed that the times between the occurrences of the earthquakes are approximately exponentially distributed with a mean of nearly 12 days for the first 67 occurrences (which provides 66 time-between-events intervals) and gives λ_0=1/(12*24)=0.0034722: We assume this to be the specified value of the rate parameter. We follow Steps 1-7 to obtain the design parameters of the ATS-unbiased tr-chart with or without runs rules (see Section 3) for a fixed nominal value of ATS_0=100,000 hours: This means that a false alarm is expected on the average, every 100000*0.0034722=347 charting points, that is, we expect an earthquake of magnitude 1.0 or higher every 11 years under normal circumstances. First, for r=1, we obtain the lower and upper control limits for the basic ATS-unbiased t1-chart, which are equal to LCL=0.63 and UCL=2093.69, respectively. In a similar manner,* "the values of the control limits for the ATS-unbiased t1-chart with the {1/1,M:3/4} scheme, are equal to LCL=31.36 and UCL=1943.22, respectively". *Figure 1* (our figure 23, below) *shows the two sets of control limits of the two ATS-unbiased t1-chart on vertical axis shown on the logarithmic scale for better visibility. Black and solid lines are the control limits of the ATS-unbiased t1-chart without runs rules scheme and red and dashed lines are the control limits of ATS-unbiased t1-chart with the modified scheme {1/1,M:3/4}. It can be observed that the* ***ATS-unbiased t1-chart*** *with and without the runs rule scheme* ***detects a signal at the 67th point****.*

According to the Kumar et al. computations, the Control Limits should be: LCL=31.36 and UCL=1943.22, quite different from the ones of Santiago&Smith. The cause is not explained by the authors…

They go on by saying: "*Thus, we next investigate monitoring the process by considering the times to every 2nd event and using the ATS-unbiased t2-chart.*" OMISSIS….

It is interesting what we find with RIT. See Table 6 (where we used the same scale of Kumar et al.)

Table 6 Comparison of results from the paper "Statistical design of ATS…." and RIT

Type of Method	LCL	UCL	Comment
N. Kumar et al. "*t1 Chart*"	0.63	2093.69	Both LCL and UCL lower than the Scientific ones
N. Kumar et al. "*ATS–unbiased t1 Chart…*"	31.36	1943.22	LCL 17 times higher than Scientific and UCL 24% of the Scientific ones
F. Galetto **RIT**	1.835	7940.01	Scientific

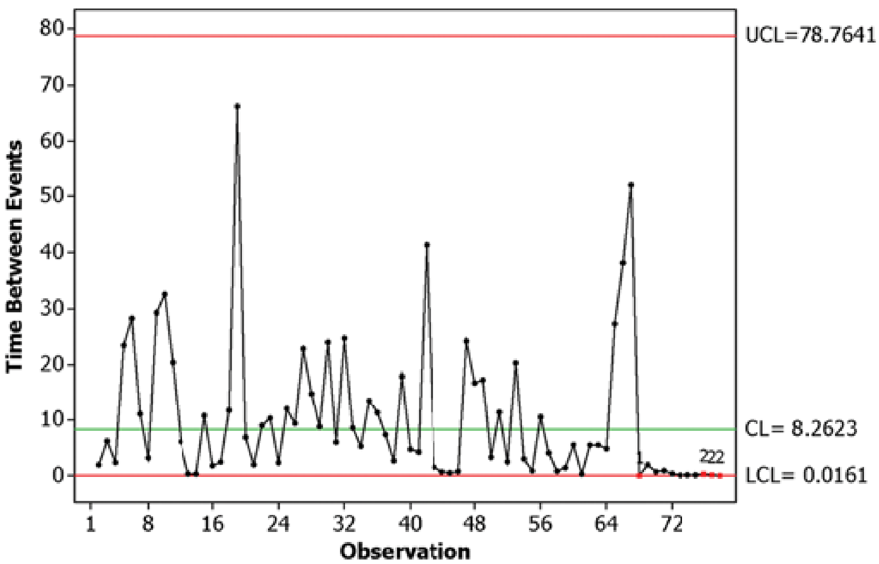
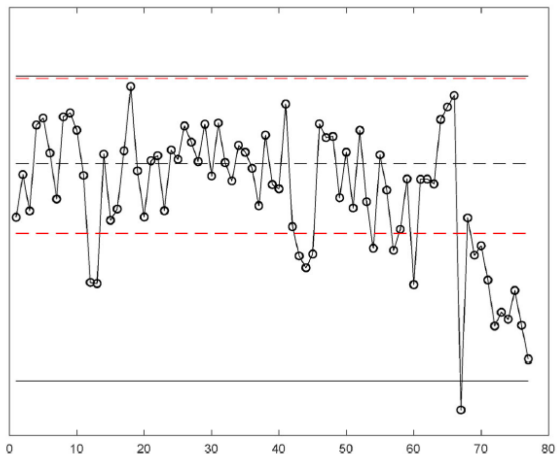


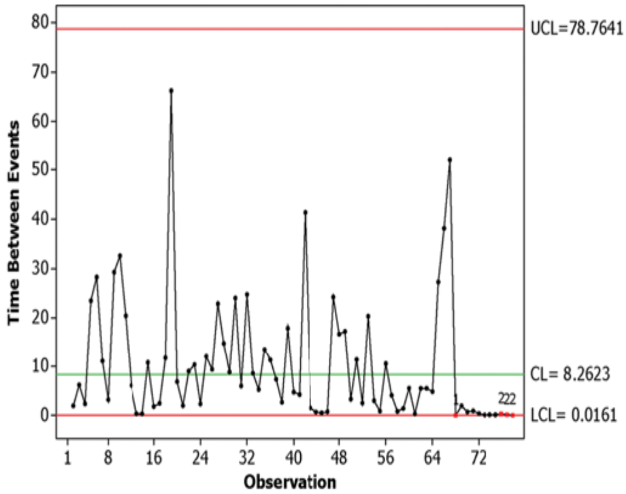
Figure 25. "Excerpt" Control Chart of "t chart of earthquakes data based on the exponential method" by Santiago& Smith, *k=1 (sample size)*

Both the methods, "*t1 Chart*" and "*ATS–unbiased t1 Chart*", from the paper "Statistical design of ATS-unbiased charts with runs rules for monitoring exponential time between events" provide wrong Control Limits.

For easy comparison we made the figure 26; both the figures are excerpt from the respective papers.

The scales are not the same (*vertical axis with the logarithmic scale* by Kumar at al. in their paper).



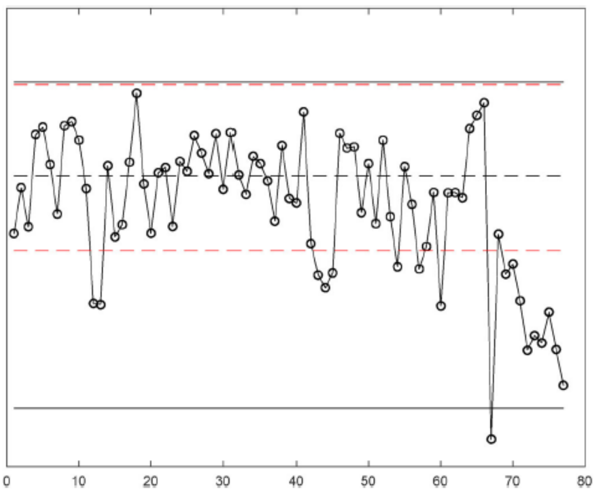


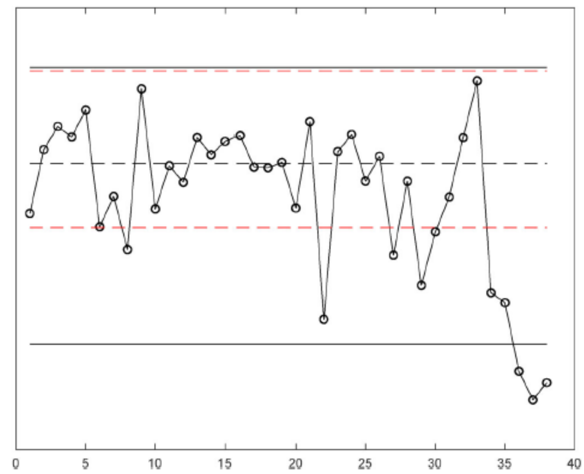
From“ATS–unbiased t1 Chart…”	From“Control charts based on the Exponential Distribution”

Figure 26. “t1 chart of earthquakes” by Kumar et al. versus Santiago& Smith, <u>*k=1 (sample size)*</u>

They say (Kumar et al.): “*It can be observed that the ATS-unbiased t1-chart with and without the runs rule scheme detects a signal at the 67th point.*” This means that the 67th point is <0.63; obviously it is also <1.835 (the LCL of Galetto).

A very strange conclusion is drawn by Kumar et al.: “*It can be observed that though the ATS unbiased t2-chart gives an OOC signal for the first time at the 36th charting point, while using the modified scheme {1/1, M : 3/4}, the chart detects an OOC signal, quite a bit sooner, at the 30th charting point.*” “*Because the ATS-unbiased t1-chart with the {1/1, M : 3/4} scheme declares an OOC signal at the 67th charting point whereas the ATS unbiased t2-chart which is based on the sum of the 59th and 60th observations, with the same scheme declares an OOC signal at the 30th point, the example supports one of the findings of this study that monitoring the times to every 2nd event (r>1) can speed up the detection of shifts in the process parameter.*”





<u>t1-chart</u>	<u>t2-chart</u>
“*ATS-unbiased t1-chart with and without the runs rule scheme detects a signal at the 67th point.*”	“*ATS unbiased t2-chart gives an OOC signal for the first time at the 36th charting point, while using the modified scheme {1/1, M : 3/4}, the chart detects an OOC signal, quite a bit sooner, at the 30th charting point.*”

Figure 27. Comparison of “t1 chart of earthquakes” and “t2 chart of earthquakes”; excerpts from Kumar et al.

Looking at figure 27 it is not clear (to us) why the authors write [right part of the figure]: "*ATS unbiased t2-chart gives an OOC signal for the first time at the **36th** charting point, while using the modified scheme {1/1, M : 3/4}, the chart detects an OOC signal, quite a bit sooner, at the **30th** charting point.*"

IF there is an OOC signal at the "***36th*** *charting point*" (for the non_modified scheme) and there is an OOC signal at the "***30th*** *charting point*" (for the Modified Scheme) why it is ***NOT OOC*** the "***22th*** *point*"<"*30th charting point*"?

The authors do not tell us. The Peer Reviewers and the editor did not find the problem!

VII. THE AVERAGE RUN LENGTH (ARL) FOR THE INDIVIDUAL CONTROL CHARTS

A way of evaluating the performance of a control charts is to compute the Average Run Length (ARL): it is the number of samples drawn from the production before finding an OOC Signal (in the control chart).

Due to the inherent variability in any process there is the probability that any process goes into OOC without any assignable cause. Therefore, a "false alarm probability α" is set for any CC: we accept that, in the long run, α% of the times the process is declared OOC, while really it is IC. Generally α=0.0027 is taken; in this case the $ARL_0=1/\alpha\approx370$ samples (the subscript 0 means ("process IC").

One problem never raised by anybody is the following:

- The estimation of ARL_0 must be based only on the stated "false alarm probability α", or must be computed from the behaviour of the "actual" used Control Chart?
- More specifically, a Shewhart Control Chart (based on the Normal distribution) and an "Individual Control Chart for TBE, I-CC_TBE (Time Between Events, exponentially distributed)", with the same "false alarm probability α" have the same computed (or estimated) ARL_0?

Only good statistical Theory can provide the solution!

Since the process behind a production process is stochastic, to compute the ARL we need the concepts about Stochastic Processes [28] (Rozanov 1975).

We have an OOC signal either when a point in the Control Chart falls out of the control limits LCL and UCL, or there is a "non-random" behaviour of the process.

We consider, here, only the first case: a point in the Control Chart falling out LCL------UCL.

Obviously, IF we WRONGLY consider that LCL------UCL (Control Interval) is the Probability Interval L------U, we have a probability problem: the time (that is, the number of samples) to go out of the probability interval L------U.

On the contrary, IF we correctly consider that LCL------UCL (Control Interval) is the Confidence Interval [which is an *equivalence class*], we have a statistical problem: the time (=the number of samples) to go out of the Confidence Interval LCL------UCL. This problem is termed "level crossing" in the stochastic literature.

If we name τ_A the random variable "time that the process X(t) takes to reach the level A", we can compute the probability $F_{\tau_A}(t)=P\{\tau_A\le t\}$ which is also given by $P\left\{\begin{matrix}\max X(s)\\ 0\le s\le t\end{matrix}\ge A\right\}=P\{\tau_A\le t\}$. Putting $\xi=\begin{matrix}\max X(s)\\ 0\le s\le t\end{matrix}$ we have the distribution $F_\xi(x)=1-P\left\{\begin{matrix}\max X(s)\\ 0\le s\le t\end{matrix}\ge x\right\}$. If we name τ_B the random variable "time that the process X(t) takes to go below the level B", we can compute the probability $F_{\tau_B}(t)=P\{\tau_B\le t\}$ which is also given by $P\left\{\begin{matrix}\min X(s)\\ 0\le s\le t\end{matrix}\le B\right\}=P\{\tau_B\le t\}$. Putting $\varphi=\begin{matrix}\min X(s)\\ 0\le s\le t\end{matrix}$ we have the distribution $F_\varphi(x)=P\left\{\begin{matrix}\min X(s)\\ 0\le s\le t\end{matrix}\le x\right\}$. Putting $\tau=min[\tau_A,\tau_B]$ we have ARL=E(τ), the mean of the RV τ.

Recalling that, the Control Limits (LCL and UCL) of the Shewhart Control Chart (based on the Normal distribution) we have that A=UCL and B=LCL.

For the "Individual Control Chart", I-CC, several books and papers use the formulae and compute the single ranges as the differences $R_i=|x_i-x_{i+1}|$, and then $\bar{R}$ as the mean of the R_i.

We performed simulations to see if those formulae provide $ARL_0\approx370$:

first, for I-CC (Individual Control Charts, based on the Normal distribution): n=20 single values x_i, 10 different first control charts CC_j, j=1,…,10, m=100 different cases C_k, k=1, …, 100, with N=500,000 simulations for any case and with each one of 10 different first control charts CC_j; we computed the various (obviously variable) $ARL_0(C_k)$ and the mean of $ARL_0(C_k)$, drawn as an horizontal line. In the Fig. 28 we see 10 horizontal lines, obviously different: "most of them" are near the theoretical value $ARL_0\approx370$.

second, for I-CC_TBE (Individual Control Charts, based on the Exponential distribution)

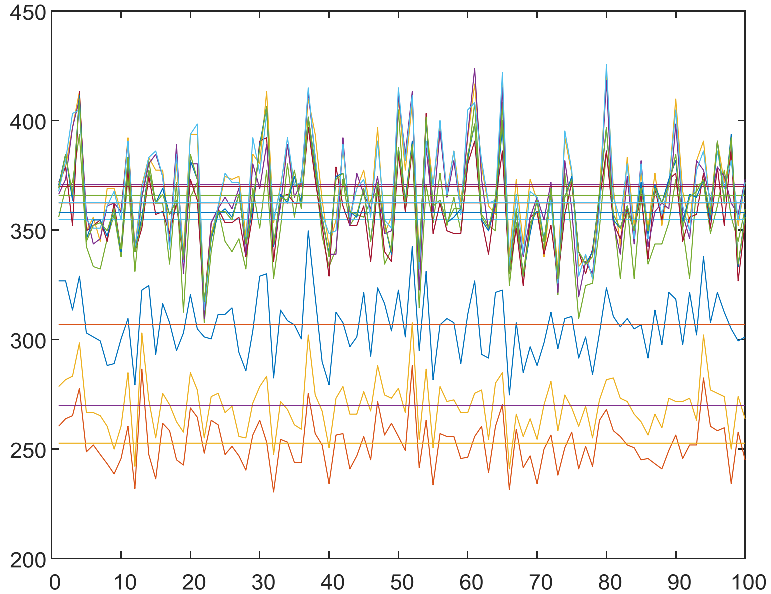


Figure 28. Values of ARL_0 (ordinate) of an ICC versus m, with Normal simulated data

So we see then that the Shewhart Individual Control Chart (based on the Normal distribution) performs as expected in the formula of Excerpt 3 (below), *whichever are the values of the parameters*, that is it depends only on the "false alarm probability α".

Would we get a similar behaviour for I-CC_TBE with TBE exponentially distributed?

The same types of calculation were performed for I-CC_TBE control charts, with the exponential distribution: see Fig. 29.

The ARL for Normal simulated data (Fig. 28) and Exponential simulated data (Fig. 29) are completely different. Why? Because of the LCL and UCL formulae! Theory explains the very different results.

See what the reader can find in "Control charts based on the Exponential Distribution", *Quality Engineering* (Santiago&Smith, 2013). The ARL is computed by the formula, (***where LCL and UCL are actually L and U***).

$$\mathrm{ARL} = \frac{1}{P(T < \mathrm{LCL}) + P(T > \mathrm{UCL})}.$$

Excerpt 3. ARL formula from (Santiago&Smith, 2013)

Remember: in the excerpt 3, the symbols LCL, UCL are NOT the Control Limits, but actually the Limits of the Probability Interval **L-----U.**

"Assuming that the process is originally modelled with T~exponential(θ_0) and that a shift in the distribution occurs such that T~exponential(θ_1), then the transient probabilities are defined as shown below:

$$P(\mathrm{LCL} < T < \mathrm{CL}) = e^{-0.001351\frac{\theta_0}{\theta_1}} - e^{-\log(2)\frac{\theta_0}{\theta_1}}$$

$$P(\mathrm{CL} < T < \mathrm{UCL}) = e^{-\log(2)\frac{\theta_0}{\theta_1}} - e^{-6.60773\frac{\theta_0}{\theta_1}}$$

$$P(\mathrm{CL} < T < \mathrm{UCL}) = e^{-0.001351\frac{\theta_0}{\theta_1}}$$

$$P(T > \mathrm{UCL}) = e^{-6.60773\frac{\theta_0}{\theta_1}}$$

Excerpt 4. Probability formulae for computing ARL from (Santiago&Smith, 2013)

To find the ARL, after the excerpt 3, Santiago&Smith use a Markov Chain with 18 states and write the formulae of excerpt 4. Remember: in the excerpt 4, the symbols LCL, UCL are NOT the Control Limits, but actually the Limits of the Probability Interval **L-----U.**

NOTICE: there is a big error in the excerpt 4 (look carefully!); Peer Reviewers and the Editor did not find it! Can you find it?

Remember: in the excerpt 4, the symbols LCL, UCL are NOT the Control Limits, but actually the Limits of the Probability Interval **L-----U**; CL is the Centre Line of the Control Chart.

Reader, see if you find something similar to the figure 22, when $\theta_0=\theta_1$, i.e. when the process remains in control.

We can understand the problem with the ARL method of Santiago&Smith by looking at figure 29 (be careful; the scales are not the “real” ones in Fig.30; see the Fig. 17):

the LCL for TBE Control Charts is quite larger than L (LCL>110 L) and the UCL is very larger than U (UCL>110 U): therefore, the probability $P\{T > UCL\} \cong 0 \ll P\{T > U\} = \alpha/2$ from which we derive $P\{T <$ $LCL>50\alpha>2PT<L=\alpha$.

Hence, for I-CC_TBE, $ARL_0(Exponential) = 1/P\{T < LCL\} \ll 1/P\{T < L\} = 1/\alpha \cong 370 \cong ARL_0(Normal)$

Santiago&Smith use L and U, the probability interval, which is not the Control Limits Interval (LCL, UCL). For the shift $\theta_0 \rightarrow \theta_1$, i.e. the process increases its mean, the interval **L-----U** (figure 29) remains of the same length U-L (because we actually do not know of the shift $\theta_0 \rightarrow \theta_1$, until we see a point above the **UCL**), while, as well, the “true” Control Limits **LCL-----UCL** remain the same as when the process was IC: this allows us to detect the shift.

KNOWLEDGE is KNOWING that

- You KNOW when you KNOW
- You DO NOT KNOW when DO NOT KNOW

Authors, Peer Reviewers, Editors, professors should learn Sound Theories and meditate on the figure 19: *Knowledge versus Ignorance, in Tools and Methods, with the Dis-quality Vicious Circle*. We do hope that the Peer Reviewers do not think (as follows): "*We do not know this author and are not familiar with his work. His claim about our formulas being wrong is not justified by any facts or material evidence. Our limits are calculated using standard mathematical statistical results/methods as is typical in the vast literature of similar papers*." Would the reader appreciate the above statement?

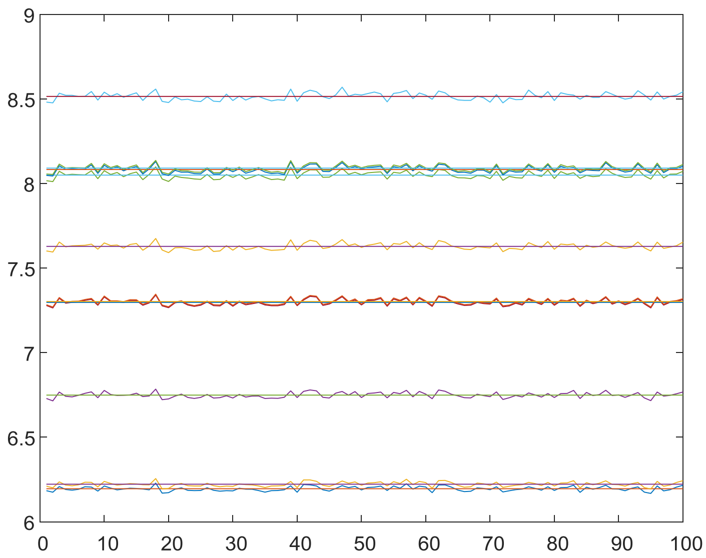


Figure 29. Values of ARL_0 (ordinate) of an I-CC_TBE versus m, with Exponential simulated data

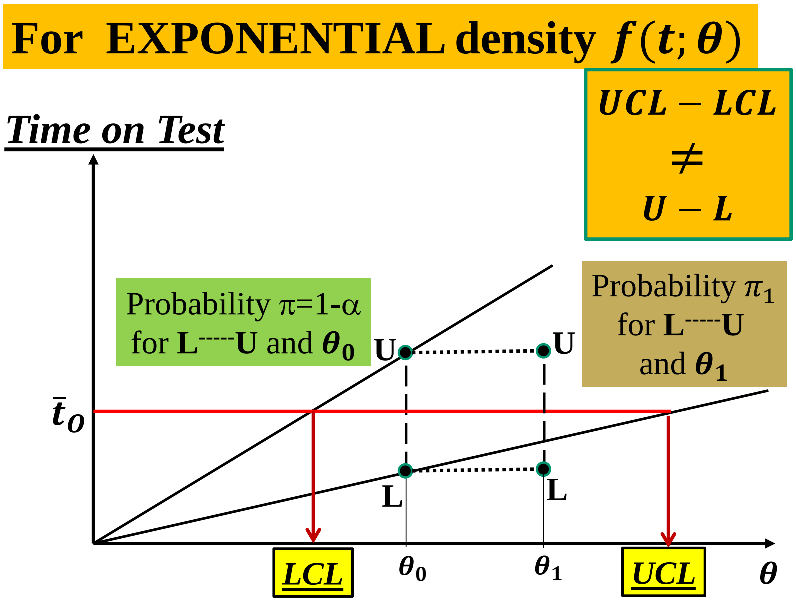


Figure 30. Graph to understand the difference about the "true (Galetto)" ARL_0 and ARL_1, due to the Control Limits **LCL----UCL** versus **L----U**, for an I-CC, with Exponential simulated data

Remember that the author wrote to the editors of *several Journals* (*Quality and Reliability….*, *Communications in Statistics…*, *Quality Technology &…*, *Journal of Quality Technology*, *Reliability Engineering &…*, *Quality Engineering*, *Journal of Statistics and …*): the letters are not yet been published: the papers are wrong and obviously the Editors cannot acknowledge that. See also the *Pentalogy*[42-43]!

VIII. DISCUSSION AND CONCLUSION

At this point, it should be clear that several Journals have been going on publishing ***wrong*** papers on I-CC_TBE [Individual Control Charts for TBE (Time Between Events)] data, exponentially distributed.

Does, now, the reader think that the statement "*The problem of monitoring TBE that follow an exponential distribution is well-defined and solved*. I do not agree that "nobody could solve scientifically the cases" has to be considered a scientific idea?

Absolutely not! This is due to lack of knowledge of the Sound Theory of the CC.

The errors are in papers published by reputed Journals, written by reputed authors and analysed by reputed Peer Reviewers, who did not find the errors: moreover, they were and are read by reputed readers, who did not find the errors (see the "*ocean full of errors …*").

None less, they are wrong…. A true disaster.

Their "formulae (wrong)" are used by the Minitab software (also R, JMP, SixPack, SAS, SPSS, ...).

The users of such software take wrong decisions based on the "wrong formulae"…

Worse than that, "the Software Management (of Minitab)" who were informed of the errors did not take any Corrective Action: a very good attitude towards Quality!

Those Journals publishing wrong papers on Control Charts for "rare events" exponentially distributed should, for future research about CC, accept the letters sent to their Editors and provide them to their Peer Reviewers.

That is the big real problem: big errors of "well reputed" people make a lot of danger, and nobody (known to the author …), but the author (FG), takes care of teaching the students to use their own brain in order not to be poisoned by incompetents.

Remember the *"XMAS Story"* with the *last statement (30 December 2022)* of the Professor *"As we see in books or mostly papers, the probability interval is consider as control interval"*… It's wrong!!! Eventually, on January 2023, with our help he got the solution, as did Galetto's students in their theses at Politecnico, more than 20 years ago.

The last document with errors we found is "*Statistical design of ATS-unbiased charts…*" published in 2022; there the data on earthquakes are used (from Santiago&Smith): the Control Limits are again wrong!

We wrote a letter to the Editors of … and to the management of Minitab Inc. about their wrong T-Charts to inform the readers of the Journals and of the software about the errors on Control Limits for TBE Charts, to avoid costly errors and decisions: the letters are not yet been published: the papers are wrong and obviously the

Editors cannot acknowledge that: the editors did not take into account the figures 19 (Tools and Methods) and 31. They did not used*metanoia*[6] (Deming 1997).

All of them never used the FAUSTA VIA [Focus, Assess, Understand, Scientifically (and Scientifically), Test, Activate, Verify, Implement, Assure], the SPQR (Semper Paratus to Quality and Reliability) principle and the CARE (Confide in Actual and Reliable Education).

How many Statisticians, Professors, Certified Master Black Belts, practitioners, workers, students, *all over the world, learned, are learning and will learn wrong methods and took and will take wrong decisions?*

Fig. 19 shows the real problem about the Minitab (or R, or SAS, or SPSS) T Charts, the Box-plot based… and ATS-unbiased charts… methods.

At Politecnico of Milan, Professors and Students can use, Free of Charge, Minitab. How many users, there, know the T Charts drawbacks? Ignorance is flooding and overflowing (due to incompetent professionals)…, like Covid 19… Therefore, we think that we helped people to avoid being cheated by …

Since the basic rules for Control Charts are based on the "Central Limit Theorem", many "professionals" transform the data to make them "almost Normally Distributed"; this behaviour can be dangerous as we showed before.

If the reader considers that the author asked many [>>50] ′′′Statisticians and Certified Master Black Belts and Minitab users (you can find them in various forums such as ReasearchGate, iSixSigma, Academia.edu, Quality Digest, … and in several Universities)′′′ and *nobody* could solve scientifically the cases, he has the dimension of the problem.

The author hopes that the Peer Reviewers of this paper have better knowledge than the discussants (in the various forums, iSixSigma, Research Gate and in the "*ocean full of errors ...*"…), otherwise he risks to be passed off…

In spite of all these proofs, the discussant who suggested the paper of J. Smith did not believe to the evidence (see the "ocean …"). He raised the problem that it could happen only by chance: he believed only in simulations (as do all who do not know Theory)! After ten millions of simulations F. Galetto got that T Charts (Minitab and in all wrong papers) were wrong 93.3% of the times!

We think that it should be enough…

Figure 31. The "epsilon Quality, driven by Intellectual hOnesty and by Gedanken Experimente".

The Fig. 31 shows the author's position in his teaching [Qualitatem Docere]: the "epsilon Quality, driven by **I**ntellectual h**O**nesty and by **G**edanken **E**xperimente".

It is related to *metanoia*[6] (Deming 1997) and to figure 19.

He always asked his students to *use their own Intelligence*, in order to avoid being poisoned by incompetents. He helped them with his papers presented at the HEI (Higher Education Institutions) Conferences since 1998.

The author, many times, with his documents, tried to compel several scholars to be scientific (from Galetto 1998 to Galetto 2022, in the References) [29 - 50]: he did not have success. Only Juran appreciated the author's ideas when he mentioned the paper "*Quality of methods for quality is important*" at the plenary session of EOQC Conference, Vienna [27] (Galetto 1989).

For the control charts, it came out that Reliability Integral Theory proved that the T Charts, for rare events and TBE, used in the software Minitab, R or SixPack or JMP or SAS are wrong.

So doing the author increased the h-index of authors publishing wrong papers.

We saw that data need to be analysed with suitable methods devised on the basis of Scientific Theory and not on methods in fashion, in order to generate the correct Control Charts. Reliability Integral Theory is able to deal with many distributions and then usable for many types of data and make Quality Decisions.

First, we introduced scientifically the concept of Confidence Interval (CI); secondly, we introduced the concept of Control Limits for the Control Charts and briefly presented the Shewhart Control Charts and the Individual Control Charts (I-CC); thirdly, we showed the correct control limits of charts with exponentially distributed data, with the applications dealt in the literature and saw the Minitab wrong calculations for the T Chart and we showed how RIT (Reliability Integral Theory) compute correctly the Control Limits for I-CC_TBE (I-CC for Time Between Events, exponentially distributed); fourthly, we showed several wrong cases taken form the literature; finally, we showed how to compute the *correct* ARL for I-CC.

We showed various cases (from books and papers) where errors were present due to the lack of knowledge of a Sound Theory of Control Charts and of RIT.RIT allows the scholars (managers, students, professors) to find sound methods also for the ideas shown by Wheeler in his Quality Digest documents.

The author has been always fond of Quality in every his activity (see figure 31); for that reason he wrote several papers and books showing scientific methods versus many wrong methods and presented them in several national and International Conferences: he wanted to diffuse Quality [29 - 50].The truth sets you free!

Deficiencies in products and methods generate huge cost of DIS-quality (poor quality) as highlighted by Deming and Juran. Any book and paper is a product (providing methods). The books presenting financial considerations about reliability with wrong ideas and methods generate huge cost for the Companies using them. The methods given in our documents provide the route to avoid such costs, especially when RIT gives the right way to deal with Preventive Maintenance (risks and costs), Spare Parts Management (cost of unavailability of systems and production losses), Inventory Management, cost of wrong analyses and decisions.

In order to show the several wrong ideas and methods related to financial and business considerations about quality in several books (not given in the references) we would need at least 80 more pages in this paper: we, obviously, cannot do that. Therefore we ask the readers to look at some of the author's documents.

A last gem: few days ago the author came across the book "*Six Sigma for the next millennium, a CSSBB guide book*" where he found "I would like to *acknowledge support* from the following individuals and organizations. (*Name*) *at MINITAB* graciously allowed me to plunder examples and other riches contained in this easy-to-use, yet profoundly powerful statistical software. *MINITAB is always my weapon of choice when assessing problems from the statistical point of view*." And so on …*Minitab*…!!! *"It is necessary to understand the theory of what one wishes to do or to make."* (Deming 1986)

Any scholar should meditate on the figure 19: *Knowledge versus Ignorance, in Tools and Methods, with the Dis-quality Vicious Circle*.

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